Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

VOLUME 16, NUMBER 2 ISSUE NO. 112 FEBRUARY, 1969

Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
664	April 2-5, 1969	New York, New York	Feb. 14, 1969
665	April 18-19, 1969	Place undetermined	Feb. 14, 1969
666	April 26, 1969	Santa Cruz, California	Feb. 14, 1969
	August 25-29, 1969 (74th Summer Meeting)	Eugene, Oregon	
	January 22-26, 1970 (76th Annual Meeting)	Miami, Florida	
	August 24-28, 1970 (75th Summer Meeting)	Laramie, Wyoming	
	January 21-25, 1971 (77th Annual Meeting)	Atlantic City, New Jersey	

*Abstracts of papers to be presented in person at the meetings must be received in the headquarters office of the Society in Providence, Rhode Island, on or before these deadlines. The deadline for by-title abstracts and news items to appear in the June *Notices* will be April 25, 1969.

The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Second-class postage paid at Providence, Rhode Island, and additional mailing offices.

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred Sixty-Fourth Meeting
Americana of New York
New York, New York
April 2-5, 1969

The six hundred sixty-fourth meeting of the American Mathematical Society will be held at the Americana of New York in New York City on April 2-5, 1969.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, the following four addresses will be presented: Professor David Shale, University of Pennsylvania, will speak on "Some probabilistic ideas in mathematical physics" at 11:00 a.m. on Friday, April 4; Professor Cathleen Morawetz, New York University, will speak on "Energy flow: wave motion and geometrical optics" at 2:00 p.m. on Friday, April 4; Professor Katsumi Nomizu, Brown University, will speak on "Differential geometry of complex hypersurfaces" at 11:00 a.m. on Saturday, April 5; Professor William Browder, Princeton University, will speak at 2:00 p.m. on Saturday, April 5, his topic to be announced later. All four addresses will be presented in Georgian Ballroom A.

There will be sessions for ten-minute contributed papers both mornings and afternoons of Friday and Saturday. The deadline for receipt of papers to be placed on the program is February 14, 1969. Abstracts should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Abstract blanks can be obtained by request from the same address. There will be no provision for late papers.

SYMPOSIUM ON MATHEMATICAL ASPECTS OF ELECTRICAL NETWORK THEORY

With the anticipated support of the Air Force Office of Scientific Research and the U. S. Army Research Office (Durham), there will be a symposium on Mathematical Aspects of Electrical Network Theory on April 2 and April 3. The AMS-SIAM Committee on Applied Mathematics chose the topic of the symposium and appointed the Organizing Committee which consists of Dr. W. A. Blackwell, Dr. Frank Branin, Dr. Robert Brayton, Professor Frank Harary, and Professor Herbert S. Wilf (chairman). The hour speakers at the symposium will be Professor J. W. T. Youngs, University of California, Santa Cruz; Dr. Robert Brayton, IBM Corporation; Dr. D. C. Youla, Brooklyn Polytechnic Institute; and Professor R. J. Duffin, Carnegie-Mellon University. In addition, there will be twelve half-hour addresses. The symposium will be held in the Regency Ballroom on Wednesday, April 2, and in the Georgian Ballroom A on Thursday, April 3.

REGISTRATION

The registration desk will be in Loire Suites 4 and 5. It will be open from 9:00 a.m. to 5:00 p.m. on Wednesday through Saturday, April 2-5.

EXHIBITS

A joint book exhibit will be on display in Vendome Suite 10 from 9:00 a.m. to 5:00 p.m. on each of the four days.

ACCOMMODATIONS

Persons intending to stay at the Americana should make their own reservations with the hotel. A reservation blank and a listing of room rates will be found on page 452 of these *Notices*. It will not appear again in the April issue.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, Americana of New York, 801 7th Avenue (7th Avenue and 52nd Street), New York, New York 10019.

Leonard Gillman
Associate Secretary

Rochester, New York

Six Hunderd Sixty-Fifth Meeting
Palmer House
Chicago, Illinois *
April 18-19, 1969

The six hundred sixty-fifth meeting of the American Mathematical Society will be held at the Palmer House, Chicago, Illinois, on April 18-19, 1969. The sessions will be held in the private dining rooms on the Club Floor (one-half flight up from the fourth floor), in the Red Lacquer Room (fourth floor), and in the Monroe Room (sixth floor) of the Palmer House.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four one-hour addresses. Professor Edward R. Fadell of the University of Wisconsin will speak on Friday, April 18, at 10:45 a.m.; his topic will be "Recent developments in fixed-point theory." Professor Pesi R. Masani of Indiana University will address the Society on Friday, April 18, at 1:45 p.m.; his subject will be "The role of vector and operator valued measures in functional analysis and probability." Professor William W. Boone of the University of Illinois will speak on Saturday, April 19, at 10:45 a.m.; his talk will be entitled "The theory of decision problems in group theory: a survey." Professor François Treves of Purdue University will address the Society on Saturday, April 19, at 1:45 p.m.; his topic will be "On local solvability of linear partial differential equations." All four hour addresses will be presented in the Red Lacquer Room.

By invitation of the same committee, there will be two special sessions of selected twenty-minute papers. One of these, to be held on Friday, April 18, at 3:00 p.m., has been arranged by Professor Arunas L. Liulevicius of the University of Chicago on the subject of K-Theory and Cohomology Operations; the speakers will be Peter Hoffman, Leif Kristensen, Peter S. Landweber, J. Peter May, and Mark Mahowald. The other special session, to be held on Saturday, April 19, at 3:00 p.m., will be on the subject of Algebraic Geometry and is being arranged by Professor Maxwell A. Rosenlicht of the University of California, Berkeley, and Northwestern University; the tentative list of speakers includes Sheeram Abhyankar, Satoshi Arima, Walter L. Baily, Jr., David Hertzig, and Stephen S. Shatz. Both of these sessions will be held in the Red Lacquer Room.

There will be sessions for contributed papers on both days of the meeting. Those having time preferences for the presentation of their papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of February 14, 1969.

REGISTRATION

The registration desk will be located in Bay 1 of the West Lounge on the fourth floor of the Palmer House. This floor can be reached by escalator from the lobby. The desk will be open from 9:00 a.m. to 5:00 p.m. on Friday and from 8:30 a.m. to 3:30 p.m. on Saturday.

ACCOMMODATIONS

A list of room rates at the Palmer House and a hotel registration form can be found on page 452 of these *Notices*. Persons

*At press time location of Midwest Meeting is undecided.

intending to stay at the Palmer House should return the registration form or a reasonable facsimile to the Palmer House by April 4.

FOOD SERVICE

There are ten restaurants within the Palmer House itself. In addition, there are many other restaurants in all price ranges in the immediate vicinity of the hotel.

TRAVEL AND LOCAL INFORMATION

The Palmer House is centrally located in downtown Chicago within a five-minute walk of the Art Institute, the Auditorium, Orchestra Hall, all of the larger department stores, and most legitimate theaters.

Those coming by way of O'Hare Airport should take the airport limousine marked "Chicago Loop"; this runs every ten or fifteen minutes and costs $2 one way. Those coming by car are advised to park in the Grant Park Underground Garage which is two blocks east of the hotel. The charge is $2.40 for 24 hours. The recommended procedure is to leave the Kennedy Expressway at the Monroe Street exit, proceed east on Monroe Street, leave luggage and passengers at the Monroe Street entrance of the Palmer House, continue east on Monroe Street, turn left on Michigan Avenue, and then immediately turn right into the Grant Park Garage. Those coming by bus or train will find the Palmer House to be within a one dollar taxi ride of the rail and bus terminals.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed to American Mathematical Society, Palmer House, 17 East Monroe Street, Chicago, Illinois 60690.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

Six Hundred Sixty-Sixth Meeting
University of California
Santa Cruz, California
April 26, 1969

The six hundred sixty-sixth meeting of the American Mathematical Society will be held at the University of California, Santa Cruz, in Santa Cruz, California, on Saturday, April 26, 1969.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited hour addresses at this meeting. Professor David Harrison of the University of Oregon will lecture at 11:00 a.m. on Saturday. The title of his talk is "On quadratic forms over general fields." Professor Theodor Ganea of the University of Washington will speak at 2:00 p.m. on Saturday. His lecture is entitled "Numerical homotopy invariants and duality." There will be sessions for contributed papers on Saturday morning and afternoon. The deadline for abstracts of contributed papers to be published in the program of the meeting is February 14, 1969. However, late papers will be accepted for presentation until a few days before the meeting. All sessions of the meeting will be held in Natural Sciences I.

Registration for the meeting will begin at 8:30 a.m. on Saturday. The registration desk will be located adjacent to Room 3, Science Lecture, in Natural Sciences I.

There are numerous hotels and motels in the Santa Cruz area, including the following:

DREAM INN
175 West Cliff Drive
Rates from $15.50 up

PASATIEMPO INN
555 Los Gatos Highway 17
Rates from $10.00 up

A complete list of motels and hotels near

Santa Cruz can be obtained from the Santa Cruz Convention and Visitors Bureau, Santa Cruz, California 95060. Reservations should be made directly with the preferred motel or hotel.

Cafeteria style luncheon will be available at the University on Saturday at a cost of approximately $0.85.

The University of California, Santa Cruz, can be reached by automobile, following Route 17 into Santa Cruz. This leads to Route 1 (Mission Street). Follow Mission Street to Bay Street, and turn right up the hill to the main entrance of the University. Then follow the circular drive through the campus to Natural Sciences I. Parking is available nearby.

Persons coming to the meeting from cities outside the northern California area can obtain air transportation to San Jose, Oakland, or San Francisco, and rent a car to drive to Santa Cruz. The Oakland and San Francisco airports are approximately 60 miles from Santa Cruz, while the San Jose Municipal Airport is about 30 miles from Santa Cruz.

R. S. Pierce
Associate Secretary

Seattle, Washington

1969 SUMMER SEMINAR CANCELED

The sixth AMS summer seminar which was to be held on the campus of Rensselaer Polytechnic Institute from July 7 through August 1, 1969, and which was to be co-sponsored by the American Meteorological Society, has been canceled because some of the granting agencies did not give their support.

NEWS ITEM

NEW CLASSIFICATION PROCEDURE FOR MATHEMATICAL BOOKS

Recently, the American Mathematical Society agreed to assist the Library of Congress in the subject classification of mathematical books. It is recognized that the accessibility of mathematical literature in libraries is greatly affected by the accuracy of its classification and subject analysis.

The AMS is requesting the publishers of mathematical literature to ask their authors to classify manuscripts of advanced textbooks, research monographs, and collections according to the AMS subject classification scheme currently used in the Society's Mathematical Offprint Service. In addition, the Society is asking publishers to adopt the procedure of printing the AMS classifications on the verso of the Title Page in each publication. Catalogers at the Library of Congress will then have available the AMS classification numbers assigned by authors for use as a guide in assigning the Library of Congress and Dewey Decimal classification numbers which most precisely reflect the subject content of the books and which most appropriately integrate the new books into the existing library collections.

The AMS will provide each publisher with a reasonable number of copies of the AMS classification scheme in English, French, Italian, German, and Russian. The Society will also provide a suggested format for printing the classification numbers on the verso of the Title Page of each book.

A valuable service will be performed for the mathematical community through the adoption of this classification policy. The AMS strongly urges all mathematics publishers to participate.

The International Congress of Mathematicians September 1-10, 1970

FIRST COMMUNICATION

The French Committee of mathematicians wishes to inform you that the next International Congress of Mathematicians will meet in Nice from September 1 to September 10, 1970.

The city of Nice is located on the shores of the Mediterranean, 18 miles from the Italian border and 600 miles from Paris.

The daily sessions of the Congress will be held in the morning in the big hall of the "Palais des Exposition", in the afternoon in the various lecture halls of the University.

The morning sessions will each be devoted to two one-hour lectures on general topics; the first will be held from 9:15 to 10:15 a.m., the second from 10:45 to 11:45 a.m.

The afternoon sessions will be devoted to approximately 250 specialized talks of a duration of at most 50 minutes (which may be shortened to leave room for discussion); each will be sponsored by one of the 33 sections of the Congress, at the following hours: from 3:00 to 3:50 p.m., from 4:10 to 5:00 p.m., and from 5:20 to 6:10 p.m.

ON INDIVIDUAL COMMUNICATIONS

The Consultative Committee of the International Mathematical Union and the Organizing Committee of the Congress have agreed unanimously to eliminate the oral individual communications of the kind given at previous Congresses. However members will have the possibility of submitting a written communication--a statement of new results or of unsolved problems--screened by a subcommittee of the Organizing Committee. Communications approved will be printed by offset and will be distributed to all members of the Congress beginning on its first day.

These reports are to be typewritten, and are to be submitted in triplicate on paper furnished by the Organizing Committee and mailed with the Second Notice in April 1969. They are to be limited to one-half page (18 typewritten lines) per author, and should be mailed so that they reach the Organizing Committee no later than October, 1969.

Mathematicians wishing to present such reports should supply this information on a form which may be obtained from the Secretariat, to be mailed so that it will reach the Organizing Committee no later than March 1, 1969.

A limited number of lecture halls will be made available for groups wishing to meet on their own initiative: such meetings are expected to provide opportunities for the discussion of some of the written individual communications.

The elimination of oral communications from the regular program of the Congress will make it possible to increase greatly the number of invited addresses. It is hoped that this program change will produce an interesting Congress.

The Secretariat of the International
Congress of Mathematicians
Collège de France
11, place Marcelin-Berthelot
75-Paris-5^{o}--France

MEMORANDA TO MEMBERS

Backlog of Mathematical Research Journals

Information on this important matter is being published twice a year, in the February and August issues of the *Notices*, with the kind cooperation of the respective editorial boards.

It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. Waiting times in particular are affected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table at the bottom of this page.

Some of the columns in the table are not quite self-explanatory, and here are some further details on how the figures were computed.

Column 2. These numbers are rounded off to the nearest 50.

Column 3. For each journal, this is the estimate as of the indicated dates, of the total number of printed pages which will have been accepted by the next time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (Pages received but not yet accepted are being ignored.)

Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society's journals) and based on these factors; manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication. There is no fixed formula.

Column 5. The first quartile (Q_1) and the third quartile (Q_3) are presented to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the *Notices*. The waiting times were measured by counting the months from receipt of manuscript in final revised form to the month in which the issue was received at the Headquarters Offices. It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.

	1	2	3		4	5		
JOURNAL	No. issues per year	Approx. No. of pages per year	BACKLOG 12/31/68	6/30/68	Est. time for paper submitted currently to be published (in months)	Observed waiting time in latest published issue (in months) Q_1	M	Q_3
American J. Math.	4	1200	1014	1697	14	21	22	23
Annals of Math.	6	1200	NR*	NR*	6-8	10	10	12
Annals of Math. Stat.	6	2200	0	0	10	9	11	13
Arch. Hist. Exact Sciences	not fixed	400	0	0	5	6	7	8
Arch. Rational Mech. Anal.	not fixed	1600	0	0	4	6	6	8
Canadian J. Math.	6	1536	1218	1470	18-20	17	18	21
Duke Math. J.	4	800-850	1598	1514	18	12	18	18
Illinois J. Math.	4	700	1400	725	24	20	20	20
J. Math. Analysis and Appl.	12	NR*	NR*	1325	12	**	**	**
J. Math. and Mech.	12	1300	720	508	7	9	11	12
J. Mathematical Physics	12	NR*	NR*	NR*	NR*	9	11	12
J. Symbolic Logic	4	650	0	0	10	9	12	17
Math. of Comp.	4	1000	0	0	8	9	10	13
Michigan Math. J.	4	500	160	180	12	11	14	18
Pacific J. Math.	12	2800	2000	NR*	14	14	16	19
Proceedings of AMS	12	3600	100	1000	9	15	16	16
Quarterly of Appl. Math.	4	700	500	600	18	12	14	16
SIAM J. on Appl. Math.	6	1550	250	0	9-11	7	8	11
SIAM J. on Control	4	750	0	0	6-8	6	6	8
SIAM J. on Numer. Anal.	4	900	0	0	6-8	9	10	12
SIAM Review	4	650	0	0	6-8	7	8	9
Studies in Appl. Math.	4	360	NR*	NR*	6	***	***	***
Transactions of the AMS	12	6200	150	1900	10	17	17	17

* NR Means that no response was received to a request for information
** Dates of receipt of manuscripts not indicated in this journal
*** New journal -- no issues published yet

Policy on Reprints

At their December meeting, the Board of Trustees established a policy that no free reprints will be given to authors if publication charges are not paid, and that this policy will become effective for all papers submitted after January 31, 1969. Only those papers which are assessed page charges under current practices will be affected by this new policy. Fifty reprints will continue to be sent in all cases where publication charges are paid; but authors will be billed if they request reprints and their institutions decline to pay the charges. Free reprints will continue to be given for articles by foreign authors, papers by authors not having an institutional sponsorship or affiliation, papers resulting from invited addresses, book reviews, and other review articles. When institutional membership dues are used as a credit to offset publication charges, free reprints will be provided.

This change in policy was made necessary by the limitation on expenditures under government grants, thereby making it difficult for institutions to meet publication charges. It is not the intention of the National Science Foundation to withhold funds permanently, and institutions may find that payment of publication charges will have to be deferred only temporarily until grant funds are released. Until funds are released, however, the Society is presented with the difficult problem of meeting commitments that were made before limitations were imposed and at a time when it was expected that publication charges would be paid in the same proportional amounts as in previous years. An example of this is the increased number of pages to be published in the TRANSACTIONS and the PROCEEDINGS.

Because of this emergency that has been created in the publication of primary journals, the Trustees have reluctantly adopted this new policy.

NEWS ITEM

NATIONAL TRANSLATIONS CENTER AT JOHN CRERAR LIBRARY

The John Crerar Library has received a grant from the National Science Foundation for partial support of the National Translations Center operated by the Library. Translations prepared by government agencies, industries, societies, academic institutions, and individuals are contributed to the Center which indexes them and makes them available upon request. There is a nominal service fee for photocopying. The collection consists of over 135,000 English translations of scientific and technical periodicals. Approximately forty percent of these translations are from the Russian, and the remainder are from Western Europe and Asia. Records of translations available from other sources are also maintained. The success of this undertaking is dependent upon the participation of all those who prepare and use translated materials. For further information or inquiries on the availability of translations on file write to the National Translations Center, John Crerar Library, 35 W. 33rd Street, Chicago, Illinois 60616.

The AMS will have a representative on the Advisory Board which is being formed, this Board to consist of representatives of 15 major scientific and professional associations.

The Translations-Register Index, a semi-monthly bulletin prepared by the National Translations Center which announces new accessions and provides a quarterly cumulative index, may be ordered from Special Libraries Association, 235 Park Avenue South, New York, New York 10003.

LETTERS TO THE EDITOR

Editor, the *Notices*

Some corrections need to be made to your article in the December 1968 *Notices* concerning the current draft law situation. A local board does not need special presidential permission to draft people over 26 year of age. The Selective Service law of 1967 provides a definite order of call which each local board must follow in filling its monthly quota. These men are drawn from the list of available I-A's. I-A's over 26 are eligible, and indeed would be drafted, once the board's supply of I-A's under 26 has been exhausted (so to speak). In addition, a little known provision of the above mentioned law allows the Selective Service System, at the request of the Defense Department, to draft people from a specific age group, say for our example: 19. If such a request were ever made (it has as yet not been; but perhaps this is the "Presidential directive" your article refers to) all current I-A's, who at one time or another after October 1967 had a II-S (student) deferment, would be combined (embedded) into the requested age group (in our example 19) and drafted with this group. In this type of case it is clear that many, many people over 26 would be inducted.

The second correction concerns procedure to be followed by a registrant in appealing a decision of his local draft board. After each new notice of classification, the registrant is entitled to a personal appearance before his board to explain his case. He must file a request for such an appearance within 30 days of the mailing of the classification notice. This personal appearance is distinct from the appeal, which is made to the state appeal board. The point is the following: some local boards, upon receipt of a letter requesting an "appeal," decide that the registrant has forfeited his personal appearance and then send the file directly to the state appeal board. The registrant is thereby denied his rights in two ways: he loses a chance to influence his local board in his favor, and secondly, and perhaps more important, he loses two or three months before his notice of induction.

One final thought: state appeal board decisions are almost always unanimous and hence presidential reviews are almost always nonexistant.

Peter R. Lipow

Editors' Note: Under the new draft laws a II-S (student) deferment has been effectively abolished for graduate students. Such deferments are only available to those who commenced graduate study befor October 1967.

Editor, the *Notices*

I should like to suggest a use of the Mathematical Offprint Service (MOS) that has apparently not been widely noticed. A subscriber who teaches a specialized course could order, for the period when he is preparing the course and teaching it, all papers classified in the field. I found that in this way I got many papers relevant to a course on approximation theory, and was able to use several as part of the course. The same principle would apply to less advanced courses if MOS were to extend its coverage to expository and pedagogical articles. People who would like MOS to introduce such extended coverage should write to MOS requesting it.

R. P. Boas

NEW AMS PUBLICATIONS

SELECTED TRANSLATION—SERIES II

Volume 77

FOURTEEN PAPERS ON SERIES AND APPROXIMATION
By Balašov, Osipov, Bojanič, Tomič, Lizorkin, Vinogradov, Suetin, Berdyšev, Timan, Jastrebova, Teljakovskiĭ, Cyganok, Tumarkin, and Efimov.

272 Pages; List Price $13.60; Member Price $10.20

Volume 78

ELEVEN PAPERS ON TOPOLOGY by A. V. Černavskiĭ, A. V. Arhangel'skiĭ, A. B. Sosinskiĭ, N. V. Veličko, P. K. Belobrov, A. S. Grek, F. I. Šmidov, Wu Wen-jun and Liao Shan-dao.

256 Pages; List Price $12.80; Member Price $9.60

LECTURES IN MATHEMATICS IN THE LIFE SCIENCES

Volume 1

SOME MATHEMATICAL PROBLEMS IN BIOLOGY
Edited by Murray Gerstenhaber

122 Pages; List Price $6.10; Member Price $4.58

A new series of publications is now being offered by the American Mathematical Society. The first volume, entitled Some Mathematical Problems in Biology, is available and contains articles by such men as E. G. Leigh, R. C. Lewontin and Theodosios Pavlidis.

The text of the first volume is comprised of lectures given at a symposium held in conjunction with the December 1966 meeting of the American Association for the Advancement of Science. Sponsored by the American Mathematical Society, the meeting dealt with mathematical theories from a biological point of view.

TRANSACTIONS OF THE MOSCOW MATHEMATICAL SOCIETY

Volume 16

380 Pages; List Price $20.90; Member Price $15.68

This volume contains articles by M. S. Agranovič on Positive Boundary Value Problems; M. I. Višik and G. I. Eskin on Convolution Equations; L. R. Volevič on Pseudodifferential Operators; Ju. V. Egorov on Hypoelliptic Operators; S. A. Gal'pern and V. E. Kondrašov on The Cauchy Problem; S. N. Kružkov on Nonlinear Parabolic Equations; E. M. Landis on Theorem of de Giorgi; V. E. Kondrašov on A Radiation Problem; V. A. Kondrat'ev on Boundary Value Problems; S. G. Gindikin on The Cauchy Problem and B. R. Vaĭnberg and S. G. Gindikin on Strong Huyghens Principle.

RECENT REPRINTS

SELECTED TRANSLATIONS--Series II

Volume 3

FIVE PAPERS ON LOGIC, ALGEBRA, AND NUMBER THEORY
By Faddeev, Linnik, Malyšev, Nečaev, Valfisz, Zykov.

256 Pages; List Price $7.60; Member Price $5.70

Volume 4

EIGHT PAPERS ON ALGEBRA AND NUMBER THEORY
By Korobov, Šafarevič, Valfisz.

244 Pages; List Price $7.30; Member Price $5.48

Volume 7

TWO PAPERS ON HOMOTOPY THEORY OF CONTINUOUS MAPPINGS
By Boltyanskiĭ and Postnikov.

328 Pages; List Price $9.80; Member Price $7.35

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 6

FOUR PAPERS ON PROBABILITY
By Chung, Kac, Erdös, Fuchs, Donsker.

60 Pages; List Price $2.00; Member Price $1.50

Number 22

A GLOBAL FORMULATION OF THE LIE THEORY OF TRANSFORMATION GROUPS
By Richard Palais.

128 Pages; List Price $2.80; Member Price $2.10

Number 27

THE GENERALIZED PONTRJAGIN COHOMOLOGY AND RINGS WITH DIVIDED POWERS
By Emery Thomas.

84 Pages; List Price $2.20; Member Price $1.65

Number 38

TORSION FREE GROUPS OF RANK TWO
By R. A. Beaumont and R. S. Pierce

44 Pages; List Price $2.00; Member Price $1.50

Number 39

TOPICS IN FOURIER AND GEOMETRIC ANALYSIS
By Victor Shapiro.

100 Pages; List Price $2.60; Member Price $1.95

Number 40

REGULAR MAPPINGS AND THE SPACE OF HOMEOMORPHISMS ON A 3-MANIFOLD
By M. E. Hamstrom.

44 Pages; List Price $1.90; Member Price $1.43

REVIEWS IN ALBEGRAIC TOPOLOGY

Norman E. Steenrod

1460 Pages; List Price $26.60; Member Price $19.95; Student Price $10.00.

In order to assist researchers and students in the fields of algebraic and differential topology, topological groups and homological algebra, Professor Norman E. Steenrod of Princeton University has compiled, edited, and classified over 6,000 reviews which have been published in Mathematical Reviews since 1940. These reviews, arranged under 290 subject headings, appear in their entirety in two new volumes now available from the American Mathematical Society.

The purpose of reprinting these reviews arranged by topic is to assist those who search the literature to find what has been done on a certain topic. According to the compiler, "The first step in using the book is to scan the list of fourteen major headings and select the heading that includes the topic in question." Additionally, the reviews listed under a single heading are arranged on the whole according to the date of publication of the paper. However, closely related papers have been brought together. A line in the margin connecting the end of one review to the beginning of the next indicates that the papers are more closely related than their occurrence under the same heading would indicate.

In addition to subject classification, the volume contains an author index, listing not only the author but the title, journal or publisher, and year of publication. Of special interest are the foreword citations which are given at the end of many reviews. These citations take note of any references made to the article at a later date.

SUMMER INSTITUTES AND GRADUATE COURSES

The following is a list of graduate courses, seminars and institutes in mathematics which are being offered in the summer of 1969 for graduate students and college teachers of mathematics.

Graduate Courses

ALABAMA

SAMFORD UNIVERSITY
Birmingham, Alabama 35209
Application deadline: July 16
Information: Dr. W. D. Peeples, Jr., Head, Mathematics Department

July 17-August 23
Math 400 - Theory of Numbers
Math 514 - Modern Geometry

UNIVERSITY OF ALABAMA (TUSCALOOSA)
University, Alabama 35486
Application deadline: April 28
Information: Robert L. Plunkett, Box 2072; or Dean of the Graduate School, Box W

July 9-July 16
Numerical Analysis I
Introduction to Complex Analysis
Introduction to Topology
*Numerical Solution of Differential Equations
*Abstract Algebra II
*Real Analysis II
*Complex Analysis II

July 17-August 22
Projective and Related Geometry I
Principles of Modern Algebra
Advanced Calculus
Theory of Statistics I
*Theory of Numbers
*Real Analysis I
*Functional Analysis I
*Algebra Seminar

*For graduate students only

ARKANSAS

ARKANSAS STATE UNIVERSITY
State University, Arkansas 72467
Application deadline: June 2
Information: J. L. Linnstaedter, Chairman, Department of Mathematics and Physics

June 2-July 11
Linear Algebra
Advanced Calculus I
Point Set Topology
Seminar (Analysis)
Number Theory

July 14-August 22
Advanced Calculus II
Projective Geometry
Functions of a Complex Variable
Seminar (Analysis)

CALIFORNIA

STANFORD UNIVERSITY
Stanford, California 94305
Application deadline: June 1
Information: George E. Forsythe, Executive Head, Computer Science Department

June 23-August 16
CS 126 Computing in the Social Sciences and Humanities
CS 136 Introduction to Algorithmic Processes
CS 239 Computer Laboratory

UNIVERSITY OF CALIFORNIA, LOS ANGELES
405 Hilgard Avenue
Los Angeles, California 90024
Application deadline: April 15
Information: Graduate Office, Department of Mathematics

June 19-September 6
Topology
Real Variables
Differential Equations
Applied Real and Complex Analysis
Tensor Analysis
Topology Seminar

COLORADO

UNIVERSITY OF COLORADO
Boulder, Colorado 80302
Application deadline: April 25
Information: S. M. Ulam, Chairman, Department of Mathematics

June 23-August 15
Math 513 - Modern Algebra I
Math 515 - Linear Algebra I
Math 531 - Introduction to Real Analysis
Math 535 - Functions of a Complex Variable I
Math 690 - Seminar in Algebra
Math 691 - Seminar in Analysis

DISTRICT OF COLUMBIA

AMERICAN UNIVERSITY
Washington, D. C. 20016
Application deadline: June 1
Information: Graduate Admissions Office American University, Washington, D.C.

July 21-September 15
Matrix Theory
Methodology I: General
Complex Variables for Applications
Managerial Statistics

FLORIDA

FLORIDA STATE UNIVERSITY
Tallahassee, Florida 32306
Application deadline: May 5
Information: Director of Admissions

June 9-August 23
STS 511 Elementary Statistical Procedures
STS 518 Finite Population Sampling
STS 546 Limit Theory of Statistics
STS 548 Linear Statistical Models
STS 597 Supervised Non-Thesis Research
STS 633 Advanced Stochastic Processes
STS 644 Advanced Topics in the Analysis of Variance

ILLINOIS

EASTERN ILLINOIS UNIVERSITY
Charleston, Illinois 61920
Application deadline: June 23
Information: A. J. DiPietro, Head, Department of Mathematics

June 24-August 18
Advanced Calculus
Modern Algebra I
Complex Variables I
Higher Geometry I
Higher Geometry II
Computer Techniques
History of Mathematics

SOUTHERN ILLINOIS UNIVERSITY
Carbondale, Illinois 62901
Application deadline: March 29 for admission to Graduate School
Information: Professor John M. H. Olmsted, Chairman, Department of Mathematics

June 17-August 29
Algebraic Structures
Linear Algebra
Seminar in Algebra
Seminar in Analysis
Seminar in Topology
Seminar in Probability and Statistics

UNIVERSITY OF ILLINOIS
Urbana, Illinois 61801
Application deadline: May 15 for admission to the Graduate College
Information: Franz E. Hohn, Mathematics Department, Altgeld Hall

June 16-August 9
Mathematics for Elementary Teachers
Introduction to Numerical Analysis
Thesis and Reading Course
The Theory of Sets and the Real Number System
Topics in Geometry
Selected Mathematics Topics for Secondary School Teachers
Linear Trans. and Matrices
Introduction to Higher Algebra, I
Introduction to Higher Algebra, II
Introduction to Set Theory and Topology
Introduction to Algebraic Topology
Advanced Calculus
Differential Equations and Orthogonal Functions
Complex Variables and Applications
Introduction to Higher Analysis-Real Variables
Introduction to Higher Analysis-Complex Variables
Topics in Pure Mathematics
Elementary Theory of Numbers, I
Theory of Probability, I
Advanced Statistics, I
Mathematical Methods in Engineering and Science
Second Course in Abstract Algebra, I
Second Course in Abstract Algebra, II
Theory of Rings
Logical Foundations of Mathematics
Elementary Geometry from a Modern Viewpoint
General Topology
Theory of Functions of a Complex Variable, I
Real Analysis, I
Real Analysis, II
Partial Differential Equations
Banach Spaces
Mathematical Methods of Physics, I
Reading Course
Thesis Research

INDIANA

BALL STATE UNIVERSITY
Muncie, Indiana 47306
Application deadline: June 1
Information: Duane E. Deal, Acting Head, Department of Mathematics

June 16-July 18
Modern Algebra
Statistics I
Computer Programming & Systems I
Higher Geometry
Elements of Analysis
Complex Variables I

July 21-August 22
Introduction to Advanced Algebra
Theory of Numbers
Theory of Statistics II
Computer Programming & Systems II
Non-Euclidean Geometry
Advanced Calculus I
Complex Variables II

PURDUE UNIVERSITY
Lafayette, Indiana 47907
Application deadline: May 1
Information: Professor M. S. Webster
Division of Mathematical Sciences

June 16-August 6
Advanced Calculus
Linear Analysis
Functions of a Complex Variable I
Principles of Analysis I & II
Abstract Algebra
Elementary Topology
Elements of Stochastic Processes
Statistical Methods I & II

IOWA

IOWA STATE UNIVERSITY
Ames, Iowa 50010
Information: T. A. Bancroft, 102 Service Building

June 3-July 11
Special Topics
in Theory
in Methods
in Design of Experiments
in Design of Surveys
Seminar on the Theory of Statistics and Probability
Research

July 14-August 22
Stochastic Processes
Special Topics
in Theory
in Methods
in Design of Experiments
in Design of Surveys
Recent Developments in Statistics and Probability
Research

UNIVERSITY OF IOWA
Iowa City, Iowa 52240
Application deadline: May 15
Information: G. P. Weeg, Chairman, Computer Science Department
June 15-August 7
22C:100 Computers & Programming I
22C:101 Computer & Programming II
22C:107 Math Fundamentals for Computer Users I

KANSAS

FORT HAYS KANSAS STATE COLLEGE
Hays, Kansas 67601
Application deadline: April 15
Information: W. Toalson, Department of Mathematics

June 9-August 6
126 Introduction to Computer Programming
159 Matrices
190 Vector Analysis
290 Problems
325 Functions of a Complex Variable
360 Advanced Calculus II
399 Thesis

MASSACHUSETTS

HARVARD UNIVERSITY
Cambridge, Massachusetts 02138
Application deadline: June 16
Information: Summer School, Holyoke Center

June 30-August 16
Introduction to Mathematical Methods I (Advanced Calculus)
Introduction to Mathematical Methods II (Differential Equations)

MICHIGAN

WESTERN MICHIGAN UNIVERSITY
Kalamazoo, Michigan 49001
Information: Department of Mathematics

April 26-June 18
Differential Geometry
Algebraic Geometry
Topology
Measure and Integration
Design of Experiments
Sampling Theory
4 Seminars

June 23-August 15
Probability
Lattice Theory
Complex Analysis
2 Seminars

MISSISSIPPI

UNIVERSITY OF MISSISSIPPI
University, Mississippi 38677
Application deadline: May 10
Information: T. A. Bickerstaff, Chairman Department of Mathematics

June 9
575 - Mathematical Statistics I
513 - Number Theory I
655 - Complex Variables I
525 - Abstract Algebra I

631 - Foundations of Geometry
625 - Linear Algebra
605 - Topology

July 16
576 - Mathematical Statistics II
514 - Number Theory II
656 - Complex Variables II
526 - Abstract Algebra II
613 - Rings and Ideals
606 - Topology
639 - Projective Geometry

MONTANA

MONTANA STATE UNIVERSITY
Bozeman, Montana 59715
Information: William Swartz, Mathematics Department

June 17-August 17
Applied Advanced Calculus
Calculus for Teachers
Special Topics in Algebra
Special Topics in Analysis
Special Topics in Statistics

NEW JERSEY

TRENTON STATE COLLEGE
Trenton, New Jersey 08625
Application deadline: June 23
Information: William Hausdoerffer, Chairman, Mathematics Department

June 30-August 8
Foundations of Geometry
Mathematical Logic

NEW YORK

FORDHAM UNIVERSITY
Bronx, New York 10458
Information: Paul Levack, Dean of the Summer Session

June 30-August 8
Fundamental Concepts of Mathematics
Introduction to Number Theory
Probability & Statistics
Theory of Rings & Fields
Group Representation

July 7-August 14
Introduction to Topology

NEW YORK UNIVERSITY
University Heights, Bronx, New York 10453
Application deadline: May 15
Information: Dean, School of Engineering and Science

June 16-July 25
T63.2201 Foundations of Mathematical Analysis

July 28-September 5
T63.2201 Foundations of Mathematical Analysis

POLYTECHNIC INSTITUTE OF BROOKLYN
333 Jay Street, Brooklyn, New York 11201
Application deadline: May 15
Information: Head, Mathematics Department

June 16-July 25
Advanced Calculus I
Real Variables I
Linear Algebra I
Numerical Analysis I
Partial Differential Equations in Mathematical Physics I
Linear and Differential Equations
Probability I
Introduction to Complex Variables

August 4-September 12
Advanced Calculus II
Real Variables II
Linear Algebra II
Numerical Analysis II
Partial Differential Equations in Mathematical Physics II
Probability II
Introduction to Complex Variables

PRATT INSTITUTE
Brooklyn, New York 11205
Application deadline: May 15
Information: Mr. Albro Newton, Assistant Director of Graduate Admissions

June 9-August 1
Courses in Mathematics
Computer Science
Operations Research

ST. JOHN'S UNIVERSITY
Jamaica, New York 11432
Application deadline: June 30
Information: Richard C. Morgan, Chairman Mathematics Department

June 30
Math. 201;202 Basic Analysis MTWTh 6:20 - 8:40 p.m.
Math. 307;308 Theory of Functions of a Real Variable Daily 10:30 - 12:20 a.m.

STATE UNIVERSITY OF NEW YORK AT ALBANY
1400 Washington Avenue
Albany, New York 12203
Application deadline: June 30
Information: Dr. V. F. Cowling, ES 110

July 1-July 23
Mat 412 Complex Variables
Mat 452 History of Mathematics
Mat 456 Foundations of Mathematics
Mat 517 Introduction to Functional Analysis
Mat 610a Real Analysis

Mat 681 Seminar in Complex Analysis
Mat 682 Seminar in Algebra
Mat 687 Seminar in Summability Theory

July 24-August 14
Mat 412 Complex Variables
Mat 452 History of Mathematics
Mat 456 Foundations of Mathematics
Mat 517 Introduction to Functional Analysis
Mat 610a Real Analysis
Mat 681 Seminar in Complex Analysis
Mat 682 Seminar in Algebra
Mat 687 Seminar in Summability Theory

NORTH CAROLINA

APPALACHIAN STATE UNIVERSITY
Boone, North Carolina 28607
Application deadline: May 25
Information: The Graduate School, Appalachian State University

June 9-July 11 & July 14-August 15
*Introduction to Real Variables
Introduction to Abstract Algebra
Introduction to Topology
Real Variables
Abstract Algebra
Topology

*Each of the introductory courses listed above is a senior-graduate course; the others are graduate courses.

NORTH CAROLINA STATE UNIVERSITY
Box 5548, State College Station
Raleigh, North Carolina 27607
Application deadline: June 9
Information: Director of Summer Sessions

June 9-July 18
MA 513
MA 524

June 10-August 8
MA 622
MA 632
MA 683

July 22-August 29
MA 514
MA 517

OHIO

BOWLING GREEN STATE UNIVERSITY
Bowling Green, Ohio 43402
Application deadline: June 23; July 28
Information: Director of Summer Sessions

June 23-July 25
Linear Algebra
Advanced Calculus
Modern Geometry
Mathematical Statistics
Topology

July 28-August 29
Foundations
Topology
Modern Algebra
Numerical Analysis

MIAMI UNIVERSITY
Oxford, Ohio 45056
Information: Elwood Bohn, Chairman Department of Mathematics

June 13-July 18
Geometry
Modern Algebra
Probability and Statistics I
Real Analysis I

July 21-August 22
Topology
Linear Algebra
Probability and Statistics II
Real Analysis II

OKLAHOMA

OKLAHOMA CITY UNIVERSITY
2501 N. Blackwelder Street
Oklahoma City, Oklahoma 73106
Information: Registrar, Oklahoma City University

June 2-July 11
Numerical Analysis

July 14-August 22
*Linear Algebra

*Sophomore course, but prospective teachers may be interested.

OREGON

OREGON STATE UNIVERSITY
Corvallis, Oregon 97331
Application deadline: June 17
Information: Department of Mathematics

June 18-August 9
Mth 412 Real Analysis
Mth 414 Vector Analysis
Mth 431, 433 Principles of Geometry
Mth 447 Abstract Algebra
Mth 507 Seminar: Analysis
Mth 507 Seminar: Problems

PENNSYLVANIA

CLARION STATE COLLEGE
Clarion, Pennsylvania 16214
Application deadline: June 9, June 23, and July 22
Information: Dr. Michel G. Ossesia, Department of Mathematics

June 23-August 1
Elementary Topology
Modern Algebra I

Modern Algebra II
Modern Geometry
Abstract Algebra II (Rings and Modules)

June 9-July 18
Advanced Calculus I
Fundamentals of Analysis I

July 22-August 29
Advanced Calculus II
Fundamentals of Analysis II

PENNSYLVANIA STATE UNIVERSITY, CAPITOL CAMPUS
Middletown, Pennsylvania 17057
Application deadline: May 21
Information: Dr. B. H. Bissinger, The Capitol Campus

10 week course, Summer Session
Math 453 (Cmp. Sc. 453) Numerical Computations--Algorithms and computer methods for interpolation, integration, smoothing, error analysis. Prerequisites: Cmp. Sc. 1 or 401 or 402 or 403; Math. 44

SHIPPENSBURG STATE COLLEGE
Shippensburg, Pennsylvania 17257
Application deadline: May 15
Information: Professor Kenneth G. Washinger, Assistant Chairman

June 9-June 27
Introduction to Functions of a Complex Variable
Foundations of Algebra

June 30-August 8
Introduction to Analysis I
Advanced Topics in Mathematics
Elements of Research

August 10-August 29
Graph Theory
Modern Algebra II

UNIVERSITY OF PITTSBURGH
Pittsburgh, Pennsylvania 15213
Application deadlines: April 1 - Spring Session; June 1 - Summer Session
Information: Admissions Committee, Department of Mathematics

April 29-June 19
Math. 242 - Functional Analysis III
Math. 270 - Topology I
Math. 277 - Laplace Transform (Theory and Applications)
Math. 290 - Applied Mathematics I

June 24-August 14
Math. 222 - Analysis III
Math. 243 - Functional Analysis IV
Math. 250 - Algebra I
Math. 271 - Topology II
Math. 291 - Applied Mathematics II

April 29-August 14
Math. 255 - Lie Algebra

UNIVERSITY OF PITTSBURGH
Pittsburgh, Pennsylvania 15213
Application deadline: April, 1969
Information: Professor Orrin E. Taulbee, Department of Computer Science

April 30-August 15
Comp. Sci. 151 Computer Operating Systems
Comp. Sci. 221 Algorithmic Languages
Comp. Sci. 227 Computational Linguistics

April 30-June 20
Comp. Sci. 281 Information Handling Systems
Comp. Sci. 294 Computer Assisted Instruction

WEST CHESTER STATE COLLEGE
West Chester, Pennsylvania 19380
Application deadline: June 1
Information: Graduate Office

June 23-August 1
Complex Variable
Theory of Sets
Theory of Equations

RHODE ISLAND

RHODE ISLAND COLLEGE
Providence, Rhode Island 02908
Application deadline: June 23
Information: Graduate Office

June 23-August 1
Mathematics 531: Advanced Number Theory (Analytic)
Mathematics 541: Probability (Theoretical Foundations)

UNIVERSITY OF RHODE ISLAND
Kingston, Rhode Island 02881

Summer 1969
Computer Science 110, Introduction to Computer Science & Algorithmic Processes
Experimental Statistics 111, Statistical Methods in Research I
Computer Science 111, Computer Organization, Programming & Information Structures
Experimental Statistics 112, Statistical Methods in Research II

TENNESSEE

EAST TENNESSEE STATE UNIVERSITY
Johnson City, Tennessee 37601
Application deadline: June 1
Information: Dr. Lester C. Hartsell, Chairman Mathematics Department

June 16-July 18
Introduction to Modern Algebra
Introduction to Modern Geometry
Introduction to Analysis
Topics from Ordinary Differential Equations
Foundations and Structure of Mathematics
Algebra for Elementary Teacher

Theory of Numbers
Modern Algebra
Advanced Differential Equations
Functions of Complex Variable
Functions of Real Variables
Special Problems in Mathematics

July 21-August 22
Introduction to Modern Algebra
Introduction to Modern Geometry
Introduction to Analysis
Topics from Partial Differential Equations
Foundations and Structure of Mathematics
Geometry for Elementary Teacher
Theory of Matrices
Modern Algebra
Partial Differential Equations
Functions of Complex Variable
Functions of Real Variables
Special Problems in Mathematics

TEXAS

EAST TEXAS STATE UNIVERSITY
Commerce, Texas 75428
Application deadline: May 1
Information: Dr. Dale R. Bedgood, Head Department of Mathematics

June 2-July 11
Math. 595 Research Lit. & Tech.
Math. 501 Mathematical Statistics
Math. 516 Essentials of Statistics
Math. 518 Master's Thesis
Math. 529 Mathematics Workshop for Elementary Teachers
Math. 543 Modern Algebra
Math. 550 Modern Concepts of Algebra for Secondary Teachers

July 14-August 22
Math. 502 Mathematical Statistics
Math. 516 Essentials of Statistics
Math. 518 Master's Thesis
Math. 529 Mathematics Workshop for Elementary Teachers
Math. 544 Introduction to Modern Algebra
Math. 560 Modern Concepts of Geometry for Secondary Teachers
Math. 595 Research Lit. & Tech.

SOUTHERN METHODIST UNIVERSITY
Dallas, Texas 75222
Application deadline: April 30
Information: Dean Claude C. Albritton, Graduate Humanities & Sciences, or Paul D. Minton, Chairman, Department of Statistics

June 2-July 11
Statistics 6325 (Old No. 135) Theory of Probability
Statistics 6379 (Old No. 179) Introduction to Markov Chains

STEPHEN F. AUSTIN STATE COLLEGE
Nacogdoches, Texas 75961
Information: W. I. Layton Box 304D

June 2-July 11
Probability Theory
College Geometry
Higher Geometry
Thesis
Graduate Research

July 14-August 22
Vector Analysis
Higher Geometry
Thesis
Graduate Research

TEXAS TECHNOLOGICAL COLLEGE
Lubbock, Texas 79409
Application deadline: May 3
Information: Robert L. Poe, Associate Chairman

June 3-July 12
Function of a Complex Variable I
Modern Algebra I
Methods of Applied Mathematics I
Design of Experiments
Topology
Mathematical Statistics I

July 14-August 23
Functions of a Complex Variable II
Modern Algebra II
Methods of Applied Mathematics II
Nonparametric Statistical Inference
Numerical Analysis
Mathematical Statistics II

UTAH

BRIGHAM YOUNG UNIVERSITY
Provo, Utah 84601
Information: K. L. Hillam, Department of Mathematics

June 15-July 17 & July 20-August 21
Math 545 C - Special Topics in Algebra
Math 585 - Matrix Analysis
Math 503 - Math for Secondary School Teachers
Math 513 C - Advanced Topics in Applied Mathematics
Math 545 D - Special Topics in Analysis

VIRGINIA

VIRGINIA POLYTECHNIC INSTITUTE
Blacksburg, Virginia 24060
Application deadline: May 1
Information: Boyd Harshbarger, Department of Statistics

June 12-July 19
Response Surface Theory
Statistics for Engineers

WASHINGTON

UNIVERSITY OF WASHINGTON
Seattle, Washington 98105
Application deadline: May 15
Information: Graduate Secretary, Department of Mathematics

June 23-July 23
507 - Foundations of Mathematics
511 - Special Topics in Algebra

July 24-August 22
508 - Foundations of Mathematics
512 - Special Topics in Algebra

WEST VIRGINIA

WEST VIRGINIA UNIVERSITY
Morgantown, West Virginia 26506
Application deadline: June 16
Information: J. C. Eaves, Chairman, or I. D. Peters, Associate Chairman

June 16-July 18
Math. 379. Functional Analysis. 3 hrs.
Math. 376. Theory of Functions of a Real Variable. 3 hrs.

June 16-August 19
Math. 315. Calculus of Variations. 3 hrs.
Math. 353. Linear Algebra. 3 hrs.

July 21-August 19
Math. 377. Theory of Functions of a Real Variable. 3 hrs.

WISCONSIN

UNIVERSITY OF WISCONSIN
Madison, Wisconsin 53706
Information: Summer Sessions Office, 602 State Street

June 23-August 16
725 Linear Transformations in Hilbert Space
751 Introductory Point Set Topology
753 Introduction to Algebraic Topology
821 Advanced Topics in Real Analysis
841 Advanced Topics in Algebra

UNIVERSITY OF WISCONSIN-MILWAUKEE
Milwaukee, Wisconsin 53201
Application deadline: May 1
Information: Department of Mathematics

June 23-August 16
600-451-9. Axiomatic Geometry
600-529-2. Foundations of Real and Complex Analysis
600-531-8. Modern Algebra
600-551-6. Elementary Topology
600-555-7. Projective Geometry
600-621-7. Introduction to Analysis
600-851-0. Advanced Topics in Topology
600-857-7. Dimension Theory

WISCONSIN STATE UNIVERSITY
Eau Claire, Wisconsin 53701
Application deadline: June 16
Information: L. F. Wahlstrom, Chairman Mathematics Department

*June 16-August 8
Math 310 Fundamental Concepts of Mathematics (not for math majors)
Math 320 Statistical Analysis for Research Workers
Math 340 Logic in Elementary Mathematics (This is a short course in the last three weeks of July).

*Some undergraduate courses are offered which are open to both undergraduate and graduate such as Differential Equations, Modern Geometry and Linear Algebra.

WISCONSIN STATE UNIVERSITY
La Crasse, Wisconsin 54601
Application deadline: June 16
Information: Dr. James M. H. Erickson, Dean The Graduate College

June 16-August 8
MA 707 Real Analysis I
MA 711 Abstract Algebra

PUERTO RICO

UNIVERSITY OF PUERTO RICO
Mayaguez, Puerto Rico 00708
Application deadline: April 15
Information: Eugene A. Francis, Chairman Department of Mathematics

June 5-July 22
Mathematics of Modern Science
Vector Analysis

Summer Institutes

ARIZONA

NORTHERN ARIZONA UNIVERSITY
Flagstaff, Arizona

Rocky Mountain Regional Summer Mathematics Seminar

Dates: July 20-August 16

Arranged by: Rocky Mountain Mathematics Consortium

Sponsoring Agents: Advanced Science Seminar Projects of the National Science Foundation

Subjects Covered: Theory of Scattering in Classical and Quantum Physics and Associated Mathematical Theories (theory of perturbation of operators on Hilbert space, eigenfunction expansions, representation theory).

Other Information: Seminar lecturers will include John D. Dollard, Tosio Kato, S. T. Kuroda, Peter D. Lax, Ralph S. Phillips, Norman Shenk, and Dale Thoe.

Deadline Date: April 20

Information: Professor C. H. Wilcox, Seminar Director, Seminar on the Mathematical Theory of Scattering, Department of Mathematics, University of Arizona, Tucson, Arizona 85721.

FLORIDA

FLORIDA STATE UNIVERSITY
Computing Center
Tallahassee, Florida 32306

National Science Foundation Institute on Calculus and the Computer

Dates: June 23-August 15

Sponsoring Agency: National Science Foundation

Subjects Covered: Computer Programming with Emphasis on Applications in the Calculus Study of experimental Calculus curricula involving computer laboratory support, Elementary numerical methods

Requirements for Admission: Ph.D. in Mathematics but lacking previous significant experience in computer programming. Holds full-time position involving college teaching of Calculus. Presents evidence his institution will encourage his efforts to initiate experimental calculus sequence during 1969-70.

Deadling Date: March 10

Information: Dr. E. P. Miles, Jr., Florida State University, Computing Center, 110 Love Bldg, Tallahassee, Florida 32306.

FLORIDA STATE UNIVERSITY
Tallahassee, Florida 32306

Conference on Limit Theorems in Statistics

Dates: September 8-September 12

Sponsoring Agency: National Science Foundation

Subjects Covered: Large deviations and Bahadur efficiency, the role of information theory in large deviation theory, special forms of the central limit theorem with remainders, and detailed asymptotic distributions for order statistics.

Deadline Date: August 15

Requirements for Admission: Professional Statisticians and students at the dissertation writing stage.

Other Information: The Conference will have a principal lecturer who will give two lectures a day. Professor R. R. Bahadur, University of Chicago, has agreed to be the principal lecturer.

Information: Dr. Ralph A. Bradley, Florida State University, Department of Statistics, Tallahassee, Florida 32306.

IOWA

IOWA STATE UNIVERSITY
Ames, Iowa 50010

Seminar on the Theory of Statistics and Probability

Dates: June 3-July 11

Subjects Covered: Theory of Statistics; Probability

Information: T. A. Bancroft, 102 Service Building, Iowa State University, Ames, Iowa 50010.

MAINE

BOWDOIN COLLEGE
Brunswick, Maine 04011

Category Theory Advanced Summer Seminar

Dates: June 24-August 14

Sponsoring Agency: National Science Foundation

Subjects Covered: Course on Categories and Functors (S. Mac Lane assisted by U. Oberst). Colloquium on Category Theory and Applications (tentatively, Artin, Barr, Bass, Chase, Hartshorne, Lawvere, Linton, Tierney). Miscellaneous seminars.

Requirements for Admission: Graduate students: endorsement by Graduate Department. Postdoctoral and senior members: must be active mathematician, adequately recommended.

Deadline Date: March 11

Other Information: Several stipends are available for graduate students and a few for postdoctorals. A limited number of senior mathematicians may receive partial subsistence and travel allowances.

Information: Dan E. Christie, Department of Mathematics, Bowdoin College, Brunswick, Maine 04011.

NEW MEXICO

NEW MEXICO STATE UNIVERSITY
Las Cruces, New Mexico 88001

Seminar for College Teachers of Mathematics

Dates: June 16-August 8

Sponsoring Agency: National Science Foundation

Subjects Covered: Abstract algebra and analysis at an elementary level.

Requirements for Admission: Applicant must be a teacher in a college or junior college with a poor background in these areas of mathematics.

Deadline Date: February 15

Information: Dr. John D. Thomas, Department of Mathematical Sciences, New Mexico State University, Las Cruces, New Mexico 88001.

NEW YORK

AMERICAN MATHEMATICAL SOCIETY
P. O. Box 6248
Providence, Rhode Island 02904

Number Theory

Sixteenth Annual Summer Research Institute to be held at the State University of New York at Stony Brook

Dates: July 7-August 1

Sponsoring Agent: American Mathematical Society

Subjects Covered: Analytic Number Theory, Diophantine Equations, Algebraic Number Theory

Qualified mathematicians who wish to participate are invited to write to Dr. Gordon L. Walker, Executive Director, American Mathematical Society.

STATE UNIVERSITY OF NEW YORK AT ALBANY
1400 Washington Avenue
Albany, New York 12203

Seminar on F_k Spaces

Dates: July 21-July 25

Application by invitation only.

Information: Dr. V. F. Cowling, ES110, State University of New York, Albany, New York 12203.

VIRGINIA

VIRGINIA POLYTECHNIC INSTITUTE
Blacksburg, Virginia 24060

Advanced Seminar In Statistics

Dates: June 12-July 19

Sponsoring Agency: National Science Foundation

Subjects Covered: Advanced Topics in Statistics, Congestion Theory.

Requirements for Admission: Advanced graduate students in statistics and mathematics.

Deadline Date: May 1

Information: Dr. Boyd Harshbarger, Department of Statistics, Virginia Polytechnic Institute, Blacksburg, Virginia 24060.

WASHINGTON

WESTERN WASHINGTON STATE COLLEGE
Bellingham, Washington 98225

Summer Institute for College Teachers in Mathematics and Computer Science

Dates: June 23-August 22

Subjects Covered: Analysis, Modern Algebra, Computer Science with Applications, Differential Equations, Abstract Algebra, Mathematical Statistics, Topology, Complex Variables, Mathematical Logic & Sets, Modern Geometry, Linear Algebra

Requirements for Admission: Most applicants will be expected to have completed some graduate study in mathematics although teachers with just a bachelor's degree with superior grades may be admitted.

Deadline Date: February 21

Information: Dr. Paul Rygg, Director, Summer Institute for Junior College Mathematics Teachers, Western Washington State College, Bellingham, Washington 98225.

WEST VIRGINIA

WEST VIRGINIA UNIVERSITY
Morgantown, West Virginia 26506

Entropy in Information Theory

Dates: June 30-August 1

Subjects Covered: Special Functions, directed by Dr. H. M. Srivastava; Topology, directed by Dr. George Mitchell

Requirements for Admission: Admission to the Graduate School of West Virginia University.

Deadline Date: June 16

Other Information: Dr. R. S. Varma, Dean of the Faculty of Mathematics, University of Delhi, will hold a visiting professorship during the summer of 1969 and direct the seminar.

WISCONSIN

UNIVERSITY OF WISCONSIN
213 Van Vleck Hall
Madison, Wisconsin 53706

Summer Institute for College Teachers of Mathematics
Dates: June 23-August 15
Sponsoring Agency: National Science Foundation
Subjects Covered: Algebra--Field Theory, Lebesgue Integration, Introductory Probability
Requirements for Admission: College teachers with strong algebra background or one year of advanced calculus.
Deadline Date: February 15
Information: J. R. Smart, Summer Institute, University of Wisconsin, 213 Van Vleck Hall, Madison, Wisconsin 53706.

CANADA

UNIVERSITÉ DE MONTRÉAL
Montréal 101, Québec, Canada

Séminaire de Mathématiques Supérieures
Dates: June 30-July 25
Sponsoring Agent: Canadian Mathematical Congress
Subjects Covered: Global Analysis and Theory of Deformations
Other Information: Financial assistance available; guest speakers will include James Eels, Max Karoubi, Joseph L. Kohn, Nicholaas H. Kuiper, Masatake Kuranishi, Raghavan Narasimhan, and Donald C. Spencer.
Information: Séminaire de Mathématiques Supérieures, Université de Montréal, Case Postale 6148, Montréal, Québec, Canada.

NEWS ITEM

1969 NORDIC SUMMER SCHOOL OF MATHEMATICS

The 1969 Nordic Summer School of Mathematics will be held June 15 to July 5, 1969, at Billströmska Folkhögskolan on the island of Tjörn which is about 60 kilometers north of Göteborg. The committee for the school consists of K. E. Aubert (Oslo), S. Bundgaard (Aarhus), L. Gårding (Lund), and O. Lehto (Helsinki). Support will be provided by the Nordic Cultural Foundation.

The subject under study will be "Pseudo-differential operators with applications to index problems." Several mathematicians of international reputation have been invited to lecture, among them L. Hormander, who will lecture on pseudo-differential operators; M. Atiyah and I. M. Singer, whose subject will be index problems; and L. Boutet de Monvel, who will lecture on boundary problems. Some lectures on K-theory will also be presented.

As the 30 participants who will be attending this school will all be at the post-graduate level of study, Departments of Mathematics in the Nordic countries are being requested to encourage qualified young mathematicians to apply for admission and to support them in their preparatory studies by arranging lectures, seminars, or study groups during the spring of 1969. Lodging and meals will be provided at Billströmska Folkhögskolan, and it is expected that travel and subsistence will be available for all participants.

Further information can be obtained by writing to Professor Tord Ganelius, Nordic Summer School of Mathematics 1969, University of Göteborg, Vasaparken, 411 24 Göteborg, Sweden.

Errata to Index of Abstracts

The following two errors have been noted in the Index to Abstracts which appeared on pages 1177 to 1211 of the December 1968 issue of these *Notices*.

F. N. SPRINGESTELL found on page 1206 should read F. N. SPRINGSTEEL.

The listing under B. J. Walsh on page 1209 of the index should include Abstract 655-82, Flux in axiomatic potential theory, Page 489.

PERSONAL ITEMS

Dr. J. H. AHLBERG of the United Aircraft Corporation, East Hartford, Connecticut, has been appointed to a professorship at Brown University in the Division of Applied Mathematics.

Dr. C. D. AHLBRANDT of the University of Oklahoma has been appointed to an assistant professorship at the University of Missouri at Columbia.

Dr. B. D. ARENDT of the University of Wisconsin has been appointed to an assistant professorship at the University of Missouri at Columbia.

Dr. L. A. AROIAN of TRW Systems has been appointed to a professorship in Industrial Administration at Union College.

Dr. LUIS BAEZ-DUARTE of Caracus, Venezuela, has been appointed a lecturer at the Massachusetts Institute of Technology.

Mr. J. B. BEDNAR of Tracor Incorporated, Austin, Texas, has been appointed to an assistant professorship at the Drexel Institute of Technology.

Professor L. M. BLUMENTHAL of the University of Missouri at Columbia was recently appointed the first Luther Marion Defoe Distinguished Professor of Mathematics.

Mr. R. L. CAMPBELL of the Case Western Reserve University has accepted a position as a systems analyst with the Computer Control Corporation, Southfield, Michigan.

Professor S. G. COUNCILMAN of the University of California, Los Angeles, has been appointed to an assistant professorship at California State College at Long Beach.

Dr. R. A. DEVORE of Ohio State University has been appointed to an assistant professorship at Oakland University.

Dr. G. F. FEEMAN of Williams College has been appointed to a professorship at Oakland University.

Professor D. J. FLEMING of Lehigh University has been appointed to an assistant professorship at Clarkson College of Technology.

Professor R. M. FOSSUM of the University of Illinois has been appointed to a visiting professorship at the University of Oslo, Norway, for the academic year 1968-1969.

Dr. E. I. GREEN of Purdue University has been appointed to an assistant professorship at the University of Missouri at Columbia.

Professor D. T. HAIMO of Southern Illinois University, Edwardsville, has been appointed to a professorship at the University of Missouri at St. Louis.

Professor E. C. JOHNSEN of the University of California, Santa Barbara, will be a visiting lecturer at the University of Michigan until April 1969.

Professor J. J. KOHN of Brandeis University has been appointed to a professorship at Princeton University.

Professor G. L. KRABBE of Purdue University is on leave for the 1968-1969 academic year at the Institute de Mathematiques Pures, Saint Martin d'Heres, France.

Dr. L. C. LARSON of the University of Kansas has been appointed to an assistant professorship at St. Olaf College.

Professor O. A. LAUDAL of the University of Illinois has been appointed a lecturer and an associate research mathematician at the University of California, Berkeley.

Professor J. B. LINDER of Guilford College has been appointed to an assistant professorship at the University of North Carolina at Charlotte.

Professor M. E. LORD of Idaho State University has been appointed to an assistant professorship at the University of Texas at Arlington.

Professor J. W. MILNOR of Princeton University has been appointed to an Alfred P. Sloan Professorship at the Massachusetts Institute of Technology.

Professor H. C. MULLIKIN of the University of Wisconsin, Madison, has been appointed to an assistant professorship at Pomona College.

Dr. L. J. NACHMAN of Ohio State University has been appointed to an assistant professorship at Okland University.

Dr. STEPHEN NEWMAN of the University of Utah has been appointed to an assistant professorship at the University of Missouri at St. Louis.

Professor GERALD PETERSON of Brigham Young University has been appointed to an assistant professorship at the University of Missouri at St. Louis.

Dr. D. H. PETTEY of the University of Utah has been appointed to an assistant professorship at the University of Missouri at Columbia.

Professor LAJOS PUKANSZKY of the University of Pennsylvania has been appointed to a visiting professorship at the Massachusetts Institute of Technology for the academic year 1968-1969.

Professor E. L. ROETMAN of the Stevens Institute of Technology has been appointed to an associate professorship at the University of Missouri at Columbia.

Dr. IRWIN SCHOCHETMAN of the University of Maryland has been appointed to an assistant professorship at Oakland University.

Dr. ULRICH SCHOENWAELDER of New Mexico State University has been appointed to an assistant professorship at the University of Missouri at St. Louis.

Dr. ALAN SCHWARTZ of the University of Wisconsin, Madison, has been appointed to an assistant professorship at the University of Missouri at St. Louis.

Professor T. I. SEIDMAN of Wayne State University has been appointed to an associate professorship at Carnegie-Mellon University.

Professor D. M. SILBERGER of Western Washington State College has been appointed to an assistant professorship at Tougaloo College.

Dr. M. S. SKAFF of the University of California, Los Angeles, has been appointed to an assistant professorship at the University of Detroit.

Dr. J. J. SOPKA of the National Bureau of Standards, Boulder, Colorado, has been appointed to a professorship at the University of Texas at Arlington.

Professor H. M. STARK of the University of Michigan has been appointed to an associate professorship at the Massachusetts Institute of Technology.

Professor ROBERT STEINBERG of the University of California, Los Angeles, has been appointed to a visiting professorship at the Massachusetts Institute of Technology for the academic year 1968-1969.

Professor N. A. TSERPES of Wayne State University has been appointed to an assistant professorship at the University of South Florida.

Dr. F. J. WEYL of the National Academy of Sciences has been appointed Dean of Sciences and Mathematics at Hunter College of the City University of New York.

Professor A. M. WHITE of Harvey Mudd College is on sabbatical leave for the academic year 1968-1969, visiting at Stanford University.

PROMOTIONS

To Professor. Emory University: W. B. EVANS.

To Associate Professor. Massachusetts Institute of Technology: V. W. GUILLEMIN; McGill University: ROBERT VERMES; Pennsylvania State University: K. T. HAHN; St. Louis University: SAM LACHTERMAN; University of Texas: H. E. LACEY; Vanderbilt University: R. L. HEMMINGER; University of Wisconsin: R. A. BRUALDI; York University: R. A. SCHAVFELE.

To Assistant Professor. University of Detroit: R. D. TRAVIS.

To Research Associate. Massachusetts Institute of Technology: R. R. KALLMAN.

INSTRUCTORSHIPS

Chicago City College, Southeast Branch: J. C. FENLEY; University of Delaware: W. M. HUBBART; Illinois Institute of Technology: R. J. DETREY; Massachusetts Institute of Technology: R. F. BERGERON, B. E. CAIN, J. T. CANNON, L. J. CORWIN, D. L. FRANK, J. M. FRANKS, K. D. JOHNSON, MANUEL LERMAN, MALAYATTIL RABINDRANATHAN, K. K. UHLENBECK; Northeastern University: K. B. JOSEPHSON; Polytechnic Institute of Brooklyn: J. S. PAPADAKIS; Rutgers University: S. L. BOYLAN; St. Joseph's College: A. M. RASH; Stevens Institute of Technology: PAUL WILLIG; Tufts University: M. W. BROOKS; Wisconsin State University, Whitewater: P. R. LIPOW.

DEATHS

Dr. MOSES RICHARDSON of Brooklyn College of the City University of New York died on December 8, 1968, at the age of 57. He was a member of the Society for 35 years.

ASSISTANTSHIPS AND FELLOWSHIPS IN MATHEMATICS IN 1969-1970

The Assistantships and Fellowships listed below are in addition to those listed on pages 1065-1160 of the December 1968 issue of these *Notices*.

Type of financial assistance (with number anticipated in 1968-1969)	Stipend: Amount (dollars)	Stipend: 9 or 12 mo.	Tuition, if not included in stipend	Service Required: Hrs/Week	Service Required: Type

DISTRICT OF COLUMBIA

Catholic University of America

WASHINGTON, D.C. 20017
Raymond W. Moller, Head
Department of Mathematics
Applications must be filed by March 1, 1969

NUMBER OF DEGREES AWARDED IN 1968
Baccalaureate Degrees by Institution: 500
Baccalaureate Degrees by Department: 15
Master's Degrees by Department: 12

PH.D. Degrees awarded during last three years by field of specialization: Algebra and Number Theory, 2; Analysis and Functional Analysis, 7; Probability and Statistics, 5. TOTAL: 14

Fellowship (8)	1800-2800	9 or 12			
Teaching Assistantship (10)	2400	9		7	Teaching
Research Assistantship (2)	3600	12	$1700		
Scholarship (4)	Tuition only				

PENNSYLVANIA

University of Pittsburgh

PITTSBURGH, PENNSYLVANIA 15213
Mario Benedicty, Chairman
Department of Mathematics

NUMBER OF DEGREES AWARDED IN 1968
Baccalaureate Degrees by Department: 80
Master's Degrees by Department: 13

PH.D. Degrees awarded during last three years by field of specialization: Algebra and Number Theory, 2; Geometry and Topology, 1; Analysis and Functional Analysis, 9; Computer Science and Numerical Analysis, 2. TOTAL: 14

Fellowship (5)	2500-2800	9-12	Incl.	Research only	
Teaching Fellowship (19)	2600-4500	8-12	Incl.	6	Teaching
Teaching Assistantship (19)	2400-4200	8-12	Incl.	6	Teaching

CANADA

University of Victoria

VICTORIA, BRITISH COLUMBIA, CANADA
S. A. Jennings, Chairman
Department of Mathematics
Applications must be filed by February 21, 1969

NUMBER OF DEGREES AWARDED IN 1968
Baccalaureate Degrees by Institution: 437
Baccalaureate Degrees by Department: 35

Teaching Assistantship (12)	1400-2400	8	$450	4	Teaching
Research Assistantship (6)	2400	8	$450	4	Research

Errata

The number of PH.D. degrees awarded in mathematics at Washington State University was incorrectly listed as 7. The number of degrees by field of specialization should have been listed as follows: Algebra and Number Theory, 5; Geometry and Topology, 9; Probability and Statistics, 1; Applied Mathematics, 2. TOTAL: 17.

Ohio University, Athens, Ohio, was incorrectly listed as Ohio State University. The correct listing follows:

OHIO

Ohio University

ATHENS, OHIO 45701
Robert G. Helsel, Chairman
Department of Mathematics
Applications must be filed by March 1, 1969

NUMBER OF DEGREES AWARDED IN 1968
Baccalaureate Degrees by Institution: 2500
Baccalaureate Degrees by Department: 60
Master's Degrees by Department: 14

The department expects to initiate a PH.D. program next fall.

Type of financial assistance (with number anticipated in 1968-1969)	Stipend Amount (dollars)	Stipend 9 or 12 mo.	Tuition, if not included in stipend	Service Required Hrs/Week	Service Required Type
Fellowship (5)	2500	9	Waived		
Teaching Assistantship (25)	2500-3000	9	Waived	3-6	Teaching
Scholarship (10)			Waived		

NEWS ITEMS

ASCE SPECIALTY CONFERENCE ON PROBABILISTIC CONCEPTS AND METHODS IN ENGINEERING

The Engineering Mechanics Division of the American Society of Civil Engineers will hold a Specialty Conference on Probabilistic Concepts and Methods in Engineering at Purdue University, Lafayette, Indiana, on November 12-14, 1969. The conference is expected to include papers on fundamentals of both continuous and discrete random process theory, applied or engineering stochastics, and probability as related to decision theory.

Authors wishing to submit papers should send an abstract of approximately 500 words to ASCE-EMD Specialty Conference, School of Civil Engineering, Purdue University, Lafayette, Indiana 47907. In order to be considered for presentation, abstracts must be received by March 1, 1969. Authors need not be ASCE members, but papers should be relevant to the topic and objectives of the conference.

SEVENTH ANNUAL SYMPOSIUM ON BIOMATHEMATICS AND COMPUTER SCIENCE IN THE LIFE SCIENCES

The Seventh Annual Symposium on Biomathematics and Computer Science in the Life Sciences, sponsored by The University of Texas Graduate School of Biomedical Sciences at Houston, Division of Continuing Education, will be held in Houston, Texas, from March 26 through 28, 1969. The following topics will be considered: Quantitative and Theoretical Biology, Mathematical and Applied Statistics in Biomedical Research, Experimental Design, Bioengineering and Simulation, Information Management--Storage and Retrieval, Computer Applications, and the Man-Machine Interface.

For further information write to the Office of the Dean, The University of Texas Graduate School of Biomedical Sciences, Division of Continuing Education, P. O. Box 20367, Houston, Texas 77025.

NEWS ITEMS AND ANNOUNCEMENTS

SYMPOSIUM ON PROBLEMS IN ANALYSIS

A symposium on Problems in Analysis, in honor of Salomon S. Bochner, will be held at Princeton University on April 1-3, 1969. The symposium will be supported by the Air Force Office of Scientific Research and Princeton University. Formal lectures will be presented by L. Carleson, K. Chandrasekharan, S. S. Chern, H. Grauert, J. F. Kingman, E. Calabi, H. Furstenberg, S. Karlin, and I. E. Segal. In addition, about twenty former students of Professor Bochner will participate in informal discussions of some problems of current interest in analysis.

SEMINAR ON DIFFERENTIAL EQUATIONS AND VARIATIONAL THEORY

A seminar on Differential Equations and Variational Theory will be held at Arkansas State University, State University, Arkansas, on March 21-22, 1969. Hour addresses will be presented by the following: Professor George M. Ewing, University of Oklahoma; Professor Edward J. McShane, University of Virginia; Professor William T. Reid, University of Oklahoma; and Professor W. Roy Utz, University of Missouri (Columbia). Further information concerning this seminar may be obtained from Professor J. L. Linnstaedter, Drawer F, State University, Arkansas 72467.

In conjunction with this seminar, the Oklahoma-Arkansas Section of the Mathematical Association of America will meet at Arkansas State University. The Association will accept a limited number of contributed papers. Further information concerning the Association meeting may be obtained from Professor Harold V. Huneke, Department of Mathematics, University of Oklahoma, Norman, Oklahoma 73069.

SEMINAR OF SCATTERING THEORY

A Rocky Mountain Regional Summer Mathematics Seminar on the Mathematical Theory of Scattering will be held from July 20 to August 16, 1969, at Northern Arizona University in Flagstaff, Arizona. The seminar is being arranged by the Rocky Mountain Mathematics Consortium and is sponsored by the Advanced Science Seminar Projects of the National Science Foundation.

The subject areas of the seminar are theory of scattering in classical and quantum physics and associated mathematical theories (theory of perturbation of operators on Hilbert space, eigenfunction expansions, representation theory). Seminar lecturers will include John D. Dollard, Tosio Kato, S. T. Kurada, Peter D. Lax, Ralph S. Phillips, Norman Shenk, and Dale Thoe.

Support will be provided for a limited number of predoctoral and postdoctoral participants. Those interested in attending and/or applying for support should write for further information to Professor C. H. Wilcox, Seminar Director, Seminar on the Mathematical Theory of Scattering, Department of Mathematics, University of Arizona, Tucson, Arizona 85721. The deadline for applications is April 20, 1969.

POST DOCTORAL FELLOWSHIPS AT THE UNIVERSITY OF MARYLAND

The Department of Mathematics at the University of Maryland, as a result of an NSF Science Development Grant, has a number of Post Doctoral Fellowships available for the academic year 1969-1970. Each fellowship carries a stipend of $10,500. No teaching duties are required, but a fellow may offer a seminar on a topic of his choice. Appointments are for one year.

INTERNATIONAL SYMPOSIUM ON NONPARAMETRIC TECHNIQUES IN STATISTICAL INFERENCE

The Department of Mathematics of Indiana University at Bloomington will hold the first International Symposium on Nonparametric Techniques in Statistical Inference from June 1 to June 6, 1969, with the support of the Air Force Office of Scientific Research. The symposium will feature invited presentations on nonparametric methods in estimation theory, sequential analysis, testing of statistical hypotheses, problems of selection and ranking, robustness, and related areas. The purpose of this conference will be to provide a forum for the dissemination of recent research in these areas and the stimulation of research and scholarship through lectures of eminent scholars from the United States and abroad. Both methodology and applications will be discussed. It is planned that the proceedings will be published.

Attendance at the symposium is open to anyone interested. Inquiries regarding the symposium should be addressed to Professor Madan L. Puri, Department of Mathematics, Indiana University, Bloomington, Indiana 47401.

RUFUS OLDENBURGER AWARD

The American Society of Mechanical Engineers has established the Rufus Oldenburger Award which consists of a medal and certificate to be given annually in perpetuity to a recipient for outstanding service to the field of automatic control. Rufus Oldenburger, who is Director of the School of Mechanical Engineering at Purdue University, was the first recipient. The award was presented at a dinner meeting of the American Society of Mechanical Engineers in New York City on December 2, 1968. At the same dinner, Professor Oldenburger also received a certificate from the American Society of Mechanical Engineers for being the "Most Honored Member of the Automatic Control Division." The award citation reads, "In recognition of his eminent achievements, distinctive contributions to the field of automatic control, and service to the Society as a scholar, engineer, inventor, educator, and author."

CONFERENCE ON TOPOLOGY

Auburn University in Auburn, Alabama, is sponsoring a topology conference to be held March 13, 14, and 15, 1969. Main emphasis will be on abstract spaces and continua theory. Following is a partial list of speakers and their topics, some of which are tentative: R. D. Anderson, Topology of the Hilbert cube and its implications for infinite dimensional manifolds; H. Cook, Tree-likeness of dendroids and λ-dendroids; F. B. Jones, Homogeneous plane continua; J. W. Rogers, Jr., Continuous mappings on continua.

In addition to the invited lectures, there will be sessions for contributed papers.

NATIONAL SCIENCE FOUNDATION SCIENCE FACULTY FELLOWSHIP AWARDS

The National Science Foundation has announced the award of 212 Science Faculty Fellowships for 1969. Among the recipients were 41 mathematicians. These fellowships are awarded to provide opportunity for junior college, college, and university teachers to improve their competence as teachers of science, mathematics, or engineering.

The Science Faculty Fellows were chosen from 1,048 applicants from the 50 states, the District of Columbia, and Puerto Rico on the basis of ability indicated by letters of recommendation, professional and academic records, and other evidence of promise and attainment. Applications were evaluated by panels appointed by the Association of American Colleges, with selection being made by the Foundation.

The Foundation expects to reopen the Science Faculty Fellowship Program in August for awards to be made in December 1969.

THE PACIFIC JOURNAL OF MATHEMATICS

The Pacific Journal of Mathematics is pleased to announce that it will increase the number of pages in each issue from about 200 pages to about 230. There is to be no increase in the price.

SEMINAR ON GLOBAL ANALYSIS AND THEORY OF DEFORMATIONS

The eighth session of the "Séminaire de mathématiques supérieures" will be held at the University of Montreal from June 30 to July 25, 1969, under the sponsorship of the Canadian Mathematical Congress. The subject of this seminar will be Global Analysis and Theory of Deformations. The program will consist of seven main courses given by the following speakers: James Eels, Max Karoubi, Joseph L. Kohn, Nicolaas H. Kuiper, Masatake Kuranishi, Raghavan Narasimhan, and Donald C. Spencer. A certain number of lectures will be given by guest speakers. Registrants may make application for financial assistance to cover traveling and living expenses.

To obtain further information concerning the program and the invited lecturers, and to obtain registration forms, write to Séminaire de Mathématiques Supérieures, Université de Montréal, Case Postale 6128, Montréal 101, Québec, Canada.

CALGARY INTERNATIONAL CONFERENCE

An international conference on Combinatorial Structures and Their Applications, sponsored by the North Atlantic Treaty Organization, will be held at the University of Calgary, Calgary, Alberta, Canada, from June 2 through 14, 1969.

The following have tentatively accepted invitations to give one-hour addresses: C. Berge, G. A. Dirac, P. Erdos, D. R. Fulkerson, J. Mycielski, C. St. J. A. Nash-Williams, G. Ringel, G.-C. Rota, H. Sachs, J. J. Seidel, P. Turán. J. Edmonds in Polyhedral Combinatorics and H. Hanani in Combinatorial Designs have each agreed to give a series of five one-hour lectures. A number of additional speakers will be invited to give half-hour lectures. Several sessions will also be available to enable delegates to present papers to the conference.

Those interested in attending the conference may obtain further information by writing to Professor E. C. Milner, Mathematics Department, University of Calgary, Calgary, Alberta, Canada.

CONFERENCE ON APPROXIMATION THEORY AND ITS APPLICATIONS

The Department of Mathematics and Center for Applied Mathematics at Michigan State University announces a three day conference on Approximation Theory and its Applications to be held on March 27-29, 1969. The lectures at the conference will be Professors Lothar Collatz of the University of Hamburg, John R. Rice of Purdue University, George Temple of Oxford University, and Richard S. Varga of Case Western Reserve University. A limited amount of support for nonlecturing participants is expected.

For a program of the conference and for further information write to Gerald D. Taylor, Conference Chairman, Department of Mathematics, Michigan State University, East Lansing, Michigan 48823.

SPECIAL YEAR IN FUNCTIONAL ANALYSIS AT INDIANA UNIVERSITY

The Mathematics Department of Indiana University is making plans to hold a special year in Functional Analysis and its Applications during the 1969-1970 academic year. The special year will be in honor of the 150th anniversary of the University, which was founded in 1820.

Several visitors proficient in the field of functional analysis or its applications will be invited to spend the academic year or a shorter period in Bloomington.

Further information may be obtained by writing to Professor George Springer, Chairman, Department of Mathematics, Swain Hall East, Indiana University, Bloomington, Indiana 47401.

ABSTRACTS PRESENTED BY TITLE

During the interval from November 2, 1968, through January 3, 1969, the papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

One abstract presented by title may be accepted per person per issue of these *Notices*. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

Algebra and Theory of Numbers

69T-A18. ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. A note on integrally closed subrings.

Let D be an integral domain with identity having quotient field K. Then these conditions are equivalent: (1) Each subring of D with identity is a Euclidean domain. (2) Each subring of D with identity is integrally closed. (3) D is either an algebraic extensions field of a finite field or the quotient field of D is the prime field of characteristic zero. (Received December 2, 1968.)

69T-A19. P. J. ERDELSKY, California Institute of Technology, Pasadena, California 91109. Some characterizations of the condition number.

If $\| \ \|$ is an induced matrix norm, the condition number $\varkappa(A) = \|A\| \, \|A^{-1}\|$ for nonsingular A is equal to the limit superior of $\|AB - I\|/\|BA - I\|$ or $\|BA - I\|/\|AB - I\|$ as $B \to A^{-1}$. Also, the supremum of $\|L - R\|$ for $\|LA - I\| \leqq \epsilon$ and $\|AR - I\| \leqq \epsilon$ is $2\epsilon\varkappa(A)/\|A\|$. Hence an ill-conditioned matrix has widely differing approximate right and left inverses. The infimum of $\|B\|$ for $A + B$ singular is $\|A\|/\varkappa(A)$; hence an ill-conditioned matrix is close to a singular matrix. The limit superior of $\|B - A^{-1}\|/\|BA - I\|$ or $\|B - A^{-1}\|/\|AB - I\|$ as $B \to A^{-1}$ is $\varkappa(A)/\|A\|$; hence an ill-conditioned matrix gives large forward errors on inversion. These results can be extended to the case of condition numbers corresponding to mixed induced norms. Some of these results are related to work of Mendelsohn, Householder, Gastinel and F. L. Bauer. (Received November 12, 1968.) (Author introduced by Professor John Todd.)

69T-A20. ELBERT M. PIRTLE, JR., University of Missouri, Kansas City, Missouri 64110. A note on irredundant intersections of valuation rings.

Let R be an integral domain with 1 and quotient field L. Let F be a family of rank one, discrete valuations on L satisfying (1) $R = \bigcap \{R_v | v \in F\}$, (2) $R_v = R_{P(v)}$, $v \in F$. R is said to satisfy (#) with respect to F if for F_1, F_2 distinct subsets of F, we have $\bigcap\{R_v|v \in F_1\} \neq \bigcap\{R_v|v \in F_2\}$. R is called a K-domain if in addition to (1), (2) above, we have (3) P(v) is divisoriel [Bourbaki, Alg. Comm., Chapter 7, p. 2] for each $v \in F$. Every Krull domain is a K-domain. In view of Gilmer's Theorem 3 in J. Algebra 4 (1966), 331-340, one might conjecture that if F is a family of rank one, discrete valuations satisfying (1), (2), and if R satisfies (#) with respect to F, then R is a Krull domain. An example shows this false. We do have the following. Theorem 1. Let F be a family of rank one, discrete valuations satisfying (1), (2) above. R satisfies (#) with respect to F iff R is a K-domain. With this we can prove the following. Theorem 2. If R is a K-domain and $X_1, X_2, \ldots, X_n$ are independent indeterminates, then $R[X_1, X_2, \ldots, X_n]$ is a K-domain. (Received November 8, 1968.)

69T-A21. JOHN DAUNS and KARL H. HOFMANN, Tulane University, New Orleans, Louisiana 70118. Nilpotent groups and automorphisms.

For any field K of characteristic $\neq 2$, G^1 is the group of all matrices $\|a(i,j)\|$ with $0 \leq i, j \leq n$; $a(i,j) \in K$; $a(i,j) = 0$ for $i > j$; $a(i,i) \neq 0$, $a_{00} = 1$. Define B^1, $B_1 \triangleleft G^1$, and $A \subseteq G^1$ by $B^1 = \{\|a(i,j)\| \in G^1 \mid a(i,j) = 0, 1 \leq i < j\}$, $B_1 = \{\|a(i,j)\| \in G^1 \mid a(i,j) = 0, i < j \leq n-1\}$, and $A = \{\|a(i,j)\| \in G^1 \mid a(0,j) = 0 \text{ for } 1 \leq j \leq n\}$. Set $N_0 \equiv [G^1,G^1]$, $N_j = [N_{j-1}, N_0]$. (I) The automorphism group Aut $A \times V$ of a semidirect product of arbitrary groups A, V is related to Aut A, Aut V and crossed homomorphisms $A \to V$, $V \to A$. Under various hypotheses, automorphisms of $A \times V$ leaving $1 \times V$ invariant are investigated. (II) Aut G^1 is determined. A noninner $\sigma \in$ Aut G^1 is constructed with $\sigma(B^1) = B_1$. Modulo inner automorphisms there is only one such. (IV) All $W \triangleleft N$ with dimension $(W \cap N_j)\, N_{j+1}/N_{j+1} = 1$, $j = 0,\ldots, n-1$, are determined. Either $W \cap N_0 \cap B_1 \neq 1$ or $W \cap N_0 \cap B^1 \neq 1$. In the latter case, there are $\alpha, \beta \in K$ such that all elements of W are of the form $I + \sum\{t_jE_{0j} | j = 1,\ldots, n\} + (\alpha t_1 + \beta t_2)E_{1n} - \beta t_1 E_{2n}$. If $\beta = 0$, W is maximal normal abelian in N_0. Analogous results hold if $W \cap N_0 \cap B_1 \neq 1$. (Received October 25, 1968.)

69T-A22. KENT R. FULLER, University of Iowa, Iowa City, Iowa 52240. Double centralizers of injectives and projectives over artinian rings.

Let R be a left artinian ring. If M is an R-module, let E(M) be the injective hull of M and let T(M) = M/rad(M). A left injective E and a right projective F form a pair with basic idempotent $f \in R$ in case (up to isomorphism) E and E(T(Rf)) have the same indecomposable direct summands and so do F and fR. Theorem. If E and F form a pair, their double centralizers are isomorphic. Let f_0 be an idempotent in R such that $T(Rf_0)$ contains exactly one copy of each minimal left ideal in R. Then $E_0 = E(T(Rf_0))$ and f_0R are, respectively, the unique minimal faithful left injective and right projective over R. Their common double centralizer D_0 is Utumi's maximal left ring of quotients for R. By calculating the double centralizer of any pair as a subring of D_0, it is shown that Theorem. A faithful left injective E (right projective F) has the double centralizer property over R iff $\mathrm{Ext}^1(S,R) = 0$ for every simple left module that does not appear in T(Rf). Theorem. Over R, the following are equivalent: (a) Every faithful left injective and every faithful right projective has the double centralizer property. (b) E_0 (resp., f_0R) has the double centralizer property. (c) $\mathrm{Ext}^1(S,R) = 0$ if ${}_RS$ is simple and $S \not\subseteq R$. (Received November 15, 1968.)

69T-A23. T. J. HEAD, University of Alaska, College, Alaska 99701. Tensor products and maximal subgroups of commutative semigroups.

Let X and Y be subsemigroups of A and B. Let $i : X \to A$ and $j : Y \to B$ be the inclusion maps. We say that $X \otimes Y$ is imbedded in $A \otimes B$ if $i \otimes j : X \otimes Y \to A \otimes B$ is 1-1. Theorem. If G and H are maximal subgroups of commutative semigroups A and B, then $G \otimes H$ is imbedded in $A \otimes B$ is a maximal subgroup of $A \otimes B$. R. O. Fulp first proved that $G \otimes H$ is imbedded in $A \otimes B$ and asked if $G \otimes H$ is a maximal subgroup of $A \otimes B$. The proof of the theorem is based on Proposition 1. If X is a subsemigroup of A whose complement is an ideal and Y is a subsemigroup of B whose complement is an ideal, then $X \otimes Y$ is imbedded in $A \otimes B$ and its complement is an ideal. Proposition 2. If X is both a subgroup and an ideal of A and Y is both a subgroup and an ideal of B, then $X \otimes Y$ is imbedded in $A \otimes B$ and is both a subgroup and an ideal of $A \otimes B$. (Received November 15, 1968.)

69T-A24. CARL D. MEYER, North Carolina State University, Raleigh, North Carolina 27606. Pseudo-inverses of triangular matrices.

Let T be an n-square complex upper (lower) triangular matrix. Let $T^{(1)}$ denote any (1)-inverse for T (i.e., $TT^{(1)}T = T$). Theorem. The following statements are equivalent: (i) there exists an upper (lower) triangular (1)-inverse for T; (ii) rank(T) = rank(T^2); (iii) T is a P_r matrix; (iv) the number of nonzero diagonal elements of T is equal to the rank of T. Theorem. Let $\lambda^n + \sum_1^n \sigma_i \lambda^{n-i} = 0$ be the characteristic equation for T. If k is the index such that $\sigma_k \neq 0$ and $\sigma_{k+1} = \sigma_{k+2} = \dots = \sigma_n = 0$, then $(1/\sigma_k)(T^{k-1} + \sum_1^{k-1} \sigma_i T^{k-i-1})$ is an upper (lower) triangular (1)-inverse for T when one exists. Since T is triangular, the coefficient σ_j is just the jth symmetric function of the diagonal elements of T. Theorem. The generalized inverse $T^\dagger$ is upper (lower) triangular if and only if the entire kth row and the entire kth column of T is 0 when the kth diagonal element of T is 0. (Received November 15, 1968.)

69T-A25. ALTON RAYMOND BROWN and DAVID R. CECIL, Noth Texas State University, Denton, Texas 76203. Group axioms via closed sets.

Let G be a nonempty set and let f be a function from $G \times G$ onto G. A subset H of G is said to be closed under f if $H \neq \emptyset$ and $f(H \times H) \subseteq H$. Let $C = \{H \subseteq G | H \text{ is closed under } f\}$. Assume the following axioms concerning G, f, and C: (A1) f is associative on G, (A2) $\bigcap C$ is nonempty and finite, (A3) if $A \subseteq G$ and $H \in C$ with $A \cap H \neq \emptyset$, then there is an $H' \in C$ with $H' \subseteq f(H \times A)$, and (A4) if $\emptyset \neq A \subseteq G$ and $H \in C$, then $A \cap f(H \times A) \neq \emptyset$. Theorem. (G,f) is a group. (Received November 22, 1968.)

69T-A26. GEORGE J. RIEGER, State University of New York at Buffalo, Amherst, New York 14226. An additive problem involving primes.

Let $1 < \sigma < 12/11$; using analytical methods, Pjateckiĭ-Šapiro (Mat. Sbornik 33 (1953), 559-566) showed that the number $\pi_\sigma(x)$ of primes $p \leq x$ of the form $p = [a^\sigma]$ for natural a satisfies $\lim_{x\to\infty} (\pi_\sigma(x) x^{-1/\sigma} \log x) = 1$. Denote by $H_\sigma(n)$ the number of representations of n in the form $n = [a^\sigma] + p$ (p prime). Applying similar analytical considerations, we obtain Theorem 1. For $1 < \sigma < 12/11$ and suitable $c_1(\sigma) > 0$, we have $H_\sigma(n) > c_1(\sigma) n^{1/\sigma} (\log n)^{-1}$. Using the sieve method, we get Theorem 2. For $1 < \sigma < 2$ and suitable $c_2(\sigma) > 0$, we have $H_\sigma(n) < c_2(\sigma) n^{1/\sigma} (\log n)^{-1}$. (Received November 22, 1968.)

69T-A27. GEORGE A. GRÄTZER and H. LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Some applications of free distributive products.

Let $\mathcal{D}$ be the category of distributive lattices with 0 and 1, and $\mathcal{S}$ the category of Stone algebras (a Stone algebra is $\langle S; \wedge, \vee, *, 0, 1 \rangle$, where $\langle S; \wedge, \vee \rangle$ is a distributive lattice with 0 and 1, a* is the pseudo-complement of a, and $a^* \vee a^{**} = 1$). Let F be the functor defined by $F(\langle S; \wedge, \vee, *, 0, 1 \rangle) = \langle S; \wedge, \vee, 0, 1 \rangle$. Let S, $S_i \in \mathcal{S}$, $i \in I$, and let S be the free product of the S_i, $i \in I$ (coproduct in $\mathcal{S}$). Theorem 1. F(S) is the free product of the $F(S_i)$, $i \in I$ (coproduct in $\mathcal{D}$). Corollary (R. Balbes and A. Horn). Let $S(\mathfrak{m})$ be the free Stone algebra on $\mathfrak{m}$ generators. Then $F(S(\mathfrak{m}))$ is the free product (in $\mathcal{B}$) of m copies of $F(S(1))$. Let $L_i, M_i \in \mathcal{B}$ for $i \in I$, I finite. For $I' \subseteq I$ let $L(I')$ be the free product of L_i, $i \in I'$, and M_i, $i \in I - I'$. Theorem 2. $\Pi(L(I')|I' \subseteq I)$ is isomorphic to the free product of $L_i \times M_i$, $i \in I$. Applications. Put $L_i = M_i = 2$. Then $L_i \times M_i = 2^2 = B(1)$, the free Boolean algebra on one generator. Hence the isomorphism gives $B(n) = 2^{2^n}$. Put $L_i = 2$, $M_i = 3$. Then $L_i \times M_i = 2 \times 3 = S(1)$. The resulting isomorphism is due to R. Balbes and A. Horn. Further applications and extensions of the isomorphisms of Theorem 2 are also given. The results of R. Balbes and A. Horn are published in Duke Math. J. (Received November 21, 1968.)

69T-A28. WILLIAM M. LAMBERT, JR., University of Detroit, Detroit, Michigan 48221. Extension of Heisler's characterization of finite fields. Preliminary report. II.

Our proof of the result claimed in part I of this report (Abstract 661-6, these *Notices* 15 (1968), 1025-1026) was inadequate; it applied only to integral domains in the weakest sense, i.e. rings without zero divisors. The original result can be obtained by strengthening somewhat the topological condition of "intrinsic regularity" in part I, viz. ($\forall$ closed $C \neq \emptyset$, R)($\exists$ continuous $f : R \to R$)(f restricted to C is identically 0, but f is not identically 0). Namely, we now require strong intrinsic regularity: ($\forall$ closed C)($\forall$ $p \in r\backslash C$)($\forall u \neq 0$)($\exists$ continuous $f : R \to R$)(f restricted to C is identically 0, and $f(p) = u$). We note that by the usual uniformity argument, the condition obtained by replacing the third universal quantifier by an existential one is equivalent to intrinsic regularity in the sense of part I. Since R is topologically homogeneous, we conjecture: Intrinsic regularity and strong intrinsic regularity are equivalent. (Received November 27, 1968.)

69T-A29. RONSON J. WARNE, West Virginia University, Morgantown, West Virginia 26506. Some properties of simple I-regular semigroups.

Let S be a simple I-regular semigroup. (I-regular semigroups. I, Abstract 659-8, these *Notices* 15 (1968), 905.) We determine the translational hull $\overline{S}$ of S and use this determination to give the ideal extensions of S by a completely 0-simple (Brandt semigroup). We give a determination of the maximal group homomorphic image of S including the defining homomorphism. We note that each congruence ρ on S is a group congruence, an idempotent separating congruence, or S/ρ is a simple I-regular semigroup with fewer $\mathcal{D}$-classes than S. We determine the idempotent separating congruences--the group congruences being in an 1-1 correspondence with the normal subgroups of the maximal group homormorphic image of S. These results extend the corresponding results of [R. J. Warne, I-bisimple semigroups, Trans. Amer. Math. Soc. 130 (1968), 367-386, and R. J. Warne, Extensions of I-bisimple semigroups, Canad. J. Math. 19 (1967), 419-426]. We also determine the maximal group homomorphic image, the idempotent separating congruences, and give a trichotomy of the congruences for a simple regular ω-semigroup [W. D. Munn, Regular ω-semigroups, Glasgow Math. J. 9 (1968), 46-66]. (Received December 4, 1968.)

69T-A30. N. E. LOSEY, University of Manitoba, Winnipeg, Canada, and C. T. BENSON, University of Oregon, Eugene, Oregon 97403. On a graph of Hoffman and Singleton.

In discussing minimal regular graphs of girth 5, Hoffman and Singleton construct a regular graph G of degree 7 with 50 vertices. Let H be the Hermitian form $X^6 + Y^6 + Z^6$ over GF(25).

A polar triangle in P, the projective plane over GF(25), is a triangle where opposite point-line pairs are dual with respect to H. A claw of G is a vertex together with 3 edges. Theorem. G may be embedded in P in such a way that the claws of G are in 1-1 correspondence with the polar triangles of P. (Received December 12, 1968.)

69T-A31. OLGA TAUSSKY, California Institute of Technology, Pasadena, California 91109. Hilbert's Theorem 94. II.

See Abstract 662-25, these *Notices* 15 (1968), 1037, for Part I. It is not known in general which class of order p becomes principal in a cyclic unramified extension of degree p. Fields with class groups of type (p,p), p an odd prime, are studied and it is assumed that only one subgroup of order p becomes principal in each cyclic unramified extension. For $p = 3$ the class groups to which the fields belong cannot be the ones which become principal in all cases. This was deduced via the 3-group of the second class field (in A. Scholz and O. Taussky, Crelle J. 171 (1934)). For $p > 3$ this is now shown to be possible from the group theoretic point of view. Further, let G be the p-group of the second class field and H the subgroup of the extension. Then $H'(G,H/H')$ is investigated. (Received December 12, 1968.)

69T-A32. JOHN A. WENZEL, University of Kansas, Lawrence, Kansas 66044. On generalizations of FC-groups.

Let G be a group which is the direct product of FC-nilpotent (FC-solvable) groups G_α, $\alpha \in \Gamma$. (For the definition of FC-nilpotent, FC-solvable, FC-center and FC-series see Duguid and McLain, Proc. Cambridge Philos. Soc. 52 (1956), 391-398.) Theorem. The group G is FC-nilpotent (FC-solvable) iff, for all α in Γ, G_α is FC-nilpotent (FC-solvable) and there exists a positive integer k such that G_α is nilpotent of class less than k (G_α is solvable of derived length less than k) for all but a finite number of groups G_α. We call a group G FC-hypercentral if its ascending FC-series terminates in G. Theorem. If G is a group and H and K are normal subgroups of G which are FC-hypercentral, then HK has nontrivial FC-center. A group G is called residually FC-central if for $x \in G$ there exist normal subgroups H and K of G such that $x \in H \backslash K$ and H/K is the FC-center of G/K. Theorem. Let G be a residually FC-central group having the minimal condition for subnormal subgroups. If H is a maximal subgroup of G, then the index of H in G is finite. (Received December 12, 1968.)

69T-A33. RAYMOND BALBES, University of Missouri, St. Louis, Missouri 63121, and G. A. GRÄTZER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Injective and projective Stone algebras.

Let $\mathscr{S}$ be the category of Stone algebras $\langle S; \wedge, \vee, *, 0, 1 \rangle$ ($\langle S; \wedge, \vee \rangle$ is a distributive lattice with 0 and 1, a^* is the pseudo-complement of a, and $a^* \vee a^{**} = 1$). A Stone algebra is injective if it is injective in $\mathscr{S}$. Theorem 1. A finite Stone algebra is injective iff it is isomorphic to a direct product of two- and three-element Stone algebras (chains). A Stone algebra is a double Stone algebra if every element has a dual pseudo-complement a^+, and $a^+ \wedge a^{++} = 0$. Theorem 2. A Stone algebra is injective iff it is a complete double Stone algebra, which has a smallest dense element (a smallest a with $a^* = 0$), and in which $a^* = b^*$ and $a^+ = b^+$ imply $a = b$. (The last condition can be replaced by "there are no distinct prime ideals P_0, P_1, P_2 with $P_0 \subset P_1 \subset P_2$".) Let B be a Boolean algebra. Then $B^{[2]} = \{\langle a,b\rangle | a \leq b, a,b \in B\}$ is a Stone-algebra, where $\wedge$ and $\vee$ is performed componentwise. Theorem 3 (Representation Theorem). Every injective Stone algebra has a representation $B_0 \times B_1^{[2]}$, where B_0 and B_1 are complete Boolean algebras. This representation is unique up to isomorphism. Theorem 4. Every Stone algebra can be embedded in an injective Stone algebra. Theorem 5. A finite Stone algebra is projective if and only if it is a direct product of Stone algebras of the following type: 0 is meet-irreducible, 1 is join irreducible, the join of two meet-irreducibles is meet-irreducible. (Received December 16, 1968.)

69T-A34. THEODORE S. ERICKSON, University of Massachusetts, Amherst, Massachusetts 01002. Jordan systems in rings with involution. Preliminary report.

A generalization of I. N. Herstein's results in Lie and Jordan systems in simple rings with involution, Amer. J. Math. 78 (1956), 629-649, is sought. Chester Tsai in The prime radical of a Jordan ring has given the following definitions of a semiprime and a prime Jordan ring: Definition. A Jordan ring R is a semiprime Jordan ring if, for any ideal A of R, $AU_A = 0$ implies $A = 0$. Definition. A Jordan ring R is a prime Jordan ring if, for any ideals A and B of R, $AU_B = 0$ implies $A = 0$ or $B = 0$. Note. $yU_x = 2x \circ (x \circ y) - x^2 \circ y$. Using these definitions we have Theorem. If R is a prime associative ring of characteristic not 2 with involution *, then S = symmetric elements under * is a prime Jordan ring. Theorem. If R is a semiprime associative ring in which $2x = 0$ implies $x = 0$ with involution *, then S is a semiprime Jordan ring. We understand that W. E. Baxter has proved these same results independently. (Received November 25, 1968.) (Author introduced by Professor Wallace S. Martindale, III.)

69T-A35. ALBERT A. MULLIN, USATAC, Warren, Michigan 48089. Applications of additive prime-number theory to aesthetics. V.

The classical Japanese NAGA-UTA poem has syllable-count structures generated by an integer multiple of repetitions of the syllable-count couplet (5,7) followed, finally, by a line with 7 syllables. E.g., every classical TANKA poem has a NAGA-UTA structure (for terminology see, e.g., these *Notices* 14 (1967), 525). Definition. Let p, q, r be odd primes. By a TORA-NAGA-UTA is meant a syllable-count form of poetry with an odd prime number of lines, with a total syllable count an odd prime number and which is formed from n consecutive syllable-count couplets (p,q) and terminating in a line of poetry with r syllables. Lemma. Every TORA-NAGA-UTA poem-form is a GETANKA (loc. cit.) poem-form. Indeed, for each choice of odd primes p, q, r with r prime to p + q, there exist infinitely many TORA-NAGA-UTA poem-forms which are GETANKA poem-forms. Dirichlet's theorem is used inductively. Problem. Determine the number of essentially different TORA-NAGA-UTA poem-forms with prescribed total number of syllables. (Received December 20, 1968.)

69T-A36. KENNETH K. HICKIN and JOHN A. WENZEL, University of Kansas, Lawrence, Kansas 66044. On normal products of FC-nilpotent and FC-hypercentral groups.

For the definition of FC-series and FC-nilpotent see Franklin Haimo, Canad. J. Math. 5 (1953), 498-511. A group G is called FC-hypercentral if there is some ordinal α such that G coincides with the αth term of the FC-series. Theorem. Let H and K be normal subgroups of a group G with H and K FC-nilpotent groups of class n and m, respectively, with $n \geq m$. Then HK is a normal FC-nilpotent subgroup of G of class at most $2n + m - 1$. Theorem. Let H and K be normal subgroups of a group G with H and K FC-hypercentral groups. Then HK is an FC-hypercentral subgroup. There is a bound on the length of the FC-series for HK in terms of the lengths of the FC-series for H and for K. This bound is calculated by means of a recursion formula. Two special cases are as follows: if α is $\geq$ the lengths of the FC-series of H and K and $\alpha = \omega^{\omega^{\mu}}$ where $\mu \geq 0$, then the length of the FC-series of HK is at most $\alpha 2$; if β is $\geq$ the lengths of the FC-series of H and K and $\omega^{\beta} = \beta$, then the length of the FC-series of HK is at most $\beta 2$. (Received December 23, 1968.)

69T-A37. J. L. CHRISLOCK, University of California, Adlai E. Stevenson College, Santa Cruz, California 95060. Identities on the bicyclic semigroup.

The bicyclic semigroup $\mathcal{C}(p,q)$ is that semigroup generated by p and q subject to the relation $pq = 1$ (pq is an identity element). Theorem. $\mathcal{C}$ satisfies a nontrivial identity. In particular, $\mathcal{C}$ satisfies $xy^2xyx^2y^2x = xy^2x^2yxy^2x$. If for real numbers r_i we set $[r_1] = (r_1 + |r_1|)/2$ and $[r_1] + \dots] + r_{m-1}] + r_m] = [[r_1] + \dots] + r_{m-1}] + r_m]$, then the graph of $h(x) = [r_1 - r_2x] + \dots] + r_{2m-1} - r_{2m}x]$ is a polygonal arc. Theorem. $\mathcal{C}$ satisfies the identity $x^{s_1}y^{t_1} \dots y^{t_m}x^{s_{m+1}} = x^{\sigma_1}y^{\tau_1} \dots y^{\tau_n}x^{\sigma_{n+1}}$ if and only if $s_1 = \sigma_1$, $t_1 = \tau_1$, $t_m = \tau_n$, $s_{m+1} = \sigma_{n+1}$, $[t_1 - s_2x] + \dots] + t_{m-1} - s_mx] = [\tau_1 - \sigma_2x] + \dots] + \tau_{n-1} - \sigma_nx]$, and $[s_2 - t_2x] + \dots] + s_m - t_mx] = [\sigma_2 - \tau_2x] + \dots] + \sigma_n - \tau_nx]$ for all $x \geq 0$. We also consider the general problem of identities on simple semigroups. Theorem. Let S be a completely simple semigroup with structure group G. S satisfies a nontrivial identity if and only if G does. In fact, if G satisfies $x_{1\alpha} \dots x_{m\alpha} = x_{1\beta} \dots x_{n\beta}$ in the variables $x_1, \dots, x_p$, then S satisfies $(xx_{1\alpha}x) \dots (xx_{m\alpha}x) = (xx_{1\beta}x) \dots (xx_{n\beta}x)$ in the variables $x, x_1, \dots, x_p$. S satisfies an identity $f(x_1, \dots, x_p) = g(x_1, \dots, x_p)$ where some variables appear more times on one side than on the other if and only if it satisfies an identity of the form $x = x(xyx)^s$, $s > 0$. (Received December 23, 1968.)

69T-A38. HARVEY I. BLAU, Yale University, New Haven, Connecticut 06520. On the degree of some finite linear groups.

Let G be a finite group with a cyclic S_p-subgroup P for some prime p. Assume that G is not of type $L_2(p)$. Let K be a field of characteristic p, and suppose there is a faithful indecomposable KG-module L of K-dimension $d \leq p$ so that $|P| = p$. Let $e = |N_G(P) : C_G(P)| = (p - 1)/t$ and finally assume $p \geq 13$. Feit [Nagoya Math. J. 27 (1966), 571-584] has shown $d \geq 7p/10 - 1/2$. Here are several improvements of this bound: (1) $d \geq \max\{3(p - 1)/4, p - e\}$ (easy); (2) $d \geq p - e/2 + t - \sqrt{(p/(p-1))(4t/3)((t-2)e + 2t + 2) + t^2}$ for $e/2 > t \geq 3$; (3) if $L \in B_0(G)$, the principal p-block, then $d > 3(p - 1)/4$ (from the symmetric and skew decomposition of $L \otimes L$). The possible structures are detailed for those nonprojective indecomposable summands of $L \otimes L^*$ which have K-dimension less than 5p. (In handling those of dim > 3p we assume t > 1.) They are used to gain further improvements for small primes, including the elimination of many specific cases with $L \in B_0(G)$. (Received December 27, 1968.)

Analysis

69T-B23. MARTIN SCHECHTER, Yeshiva University, New York, New York 10033. Spectra of tensor products.

Let $X_1,\dots,X_n$ be Banach spaces and let W be the completion of the tensor product $X_1 \otimes \dots \otimes X_n$ with respect to some crossnorm. Let A_i be a bounded linear operator in X_i, and let T_i be the operator on W defined by $T_i = I_1 \otimes \dots \otimes A_i \otimes \dots \otimes I_n$, where I_k is the identity operator on X_k. Let $P(z_1,\dots,z_n)$ be a polynomial in n variables. Theorem. The spectrum of $P(T_1,\dots,T_n)$ consists of those μ such that $\mu = P(\lambda_1,\dots,\lambda_n)$, where $\lambda_k \in \sigma(A_k)$, $1 \leqq k \leqq n$, i.e. $\sigma[P(T_1,\dots,T_n)] = P[\sigma(A_1),\dots,\sigma(A_n)]$. (Received October 23, 1968.)

69T-B24. ANDRE de KORVIN and R. J. EASTON, Indiana State University, Terre Haute, Indiana 47809. Some properties of states on B*-algebras.

Let N and M be B*-algebras with identity such that $N \subset M$. A scalar valued function ρ defined on M will be called a state if ρ is linear, of norm one, positive in the sense that $\rho(x^*x) \geqq 0$, and satisfies $\rho(x^*) = \overline{\rho(x)}$. A state ρ is said to diagonalize N if $\rho(mn) = \rho(nm)$ for all $m \in M$, $n \in N$. A state ρ is said to be faithful if $\rho(x^*x) = 0$ implies $x = 0$. A collection $\{\rho_\alpha\}$ of states is said to be complete if $\rho_\alpha(x^*x) = 0$ for all α implies $x = 0$. Definition 1. A state ρ is said to be normal on N if $\rho(x^*x \sup p_\beta) = \sup \rho(x^*xp_\beta)$ for all increasing nets $\{p_\beta\}$ of projections in Z_N (center of N) with $\sup p_\beta \in Z_N$ and for all $x \in N$. Definition 2. A state ρ will be called continuously faithful on N if there exists a projection $p \neq 0$ in Z_N such that ρ is faithful on $N_p = \{pap\colon a \in N\}$ and further if whenever $\{p_\beta\}$ is an increasing net of projections in Z_N with ρ faithful on each N_{p_β} and $\sup p_\beta \in Z_N$, then ρ is faithful on $N_{\sup p_\beta}$. Theorem 1. If ρ diagonalizes M and satisfies the boundedness condition that there exists a constant $k > 0$ such that for all $x,y \in N$, $\rho(x^*xy^*y) \leqq k\rho(x^*x)\rho(y^*y)$, then for any projection $q \in Z_N$ with ρ faithful on N_q, (N_q,ρ) forms a Hilbert algebra under $(x,y) = \rho(xy^*)$. Theorem 2. If $\{\rho_\alpha\}$ is a complete set of states on M such that each ρ_α is normal on N, then each ρ_α is continuously faithful on N. (Received October 23, 1968.)

69T-B25. MALCOM S. ROBERTSON, University of Delaware, Newark, Delaware 19711. Univalent functions f(z) for which zf'(z) is spirallike.

Let α be a real number in the interval $(-\pi/2,\pi/2)$. Let $S(\alpha)$ denote the class of functions that satisfy the conditions: (1) f(z) is regular for $|z| < 1$ and normalized so that $f(0) = 0$, $f'(0) = 1$; (2) $f'(z) \neq 0$ for $|z| < 1$; (3) $\mathrm{Re}[e^{i\alpha}(1 + zf''(z)/f'(z))] > 0$ for $|z| < 1$. Then zf'(z) is univalent and spirallike for $|z| < 1$ whenever $f(z) \in S(\alpha)$. We show by an example that if $1/2 < \cos\alpha < 1$, f(z) may be a member of $S(\alpha)$ and yet not be univalent in $|z| < 1$. However, if $f(z) \in S(\alpha)$ and if $0 < \cos\alpha < x_0 = 0.2315\dots$, where x_0 is the positive root of the equation $16x^3 + 16x^2 + x - 1 = 0$, then f(z) is always univalent for $|z| < 1$. It is an open question whether f(z) is univalent in the unit disc when the range is extended to include $0 < \cos \alpha \leqq 1/2$. The case $\cos \alpha = 1$ corresponds to the situation in which $f(z)$ maps $|z| < 1$ onto a convex domain. (Received October 30, 1968.)

69T-B26. SYED M. MAZHAR, Aligarh Muslim University, Aligarh, India. On the summability $|\bar{N}, p_n|$ of Fourier series.

In this note the following theorem has been proved. Theorem. Let $f(x) \in L(-\pi,\pi)$ and be periodic with period 2π. If at every point y on the closed interval $[-\pi, \pi]$, there are a function $g_y(x)$ and a $\delta > 0$ such that (i) $g_y(x) = f(x)$ for $|x - y| < \delta$ and (ii) the Fourier series of $g_y(x)$ is summable $|\bar{N}, p_n|$, where $p_n > 0$, $\{p_n\}$ is monotonic nonincreasing and $p_n/p_{2n} < K$ for all n, then the Fourier series of $f(x)$ is summable $|\bar{N}, p_n|$. This includes, as a special case for $p_n = 1$, a result of Randels (Duke Math. J. 7(1940), 204-207) and for $p_n = 1/n$, a theorem of R. G. Varshney on $|R, \log n, 1|$ summability of Fourier series. (Received November 6, 1968.)

69T-B27. H. M. SRIVASTAVA, West Virginia University, Morgantown, West Virginia 26506. Certain formulas associated with generalized Rice polynomials.

Some recent expansions [cf. Proc. Cambridge Philos. Soc. 64 (1968), 431-434] involving the generalized Rice polynomial $H_n^{(\alpha,\beta)}[S,p,v] = \binom{\alpha+n}{n}\, {}_3F_2[-n, \alpha + \beta + n + 1, S; 1 + \alpha, p; v]$, $n \geqq 0$, are first exhibited as specialized or limiting forms of certain known results due to the author [On q-generating functions and certain formulas of David Zeitlin, Abstract 654-17, these *Notices* 15 (1968), 341; see also An extension of the Hille-Hardy formula, Math. of Comp. 23 (1969), to appear] and then extended to hold for various classes of generalized hypergeometric polynomials. The bilinear generating functions derived here involve a general class of hypergeometric functions in three arguments which the author introduced a couple of years ago [Generalized Neumann expansions involving hypergeometric functions, Proc. Cambridge Philos. Soc. 63 (1967), 425-429]. (Received November 7, 1968.)

69T-B28. W. A. KIRK, and W. D. ROYALTY, University of Iowa, Iowa City, Iowa. Fixed point theorems for certain nonlinear nonexpansive mappings. Preliminary report.

Let f be a mapping of a metric space M into itself; for $x \in M$ let $O(f^n(x)) = \{f^n(x), f^{n+1}(x),\ldots\}$, $n = 0,1,2,\ldots$. If either $\lim_{n\to\infty} \delta\, O(f^n(x)) < \delta\, O(x)$ or $\delta\, O(x) = 0$, then f is said to have diminishing orbital diameter (d.o.d.) at x. Our principal result is the following: Theorem. Suppose K is a nonempty, weakly compact, convex subset of a Banach space; let $T: K \to K$ be nonexpansive ($\|Tx - Ty\| \leqq \|x - y\|$, $x, y \in K$). If for each $x \in K$ there is an integer n, depending on x, such that T^n has d.o.d. at x, then T has a fixed point in K. That this is true for $n \equiv 1$ is a consequence of a result of Belluce-Kirk (Abstract 67T-428, these *Notices* 14 (1967), 547). It is proved for fixed $n > 1$ in Kirk [Proc. Sympos. on Nonlinear Functional Analysis, April 1968 (to appear)]. (Received November 4, 1968.)

69T-B29. JAMES WARD BROWN, University of Michigan, Dearborn Campus, and 1014 Church Street, Apt. B-2, Ann Arbor, Michigan 48104. New generating functions for classical polynomials.

The author has exhibited for the generalized hypergeometric polynomials $p_n(x) = \binom{\alpha+\beta n}{n} {}_{A+1}F_{B+1}[-n,(a);\ \alpha + 1 + (\beta - 1)n,\ (b);\ x]$ the generating functions $\sum_{n=0}^{\infty} p_n(x)t^n = [1 - u(t)]^{\alpha+1}[1 + (\beta - 1)u(t)]^{-1}\ {}_AF_B[(a);\ (b);\ xu(t)]$ and $\sum_{n=0}^{\infty} \alpha p_n(x)t^n/(\alpha + \beta n) = [1 - u(t)]^{\alpha}\ {}_{A+1}F_{B+1}[\alpha/\beta,\ (a);\ \alpha/\beta + 1,\ (b);\ xu(t)]$ where $u(t)$ is the inverse of $v(t) = -t(1 - t)^{-\beta}$. Special cases provide a variety of new generating functions for the classical polynomials containing parameters--Jacobi, Gegenbauer, and Laguerre. The derivations are based on modifications of Gould's (Duke Math. J. 28 (1961), 193-202) Vandemonde convolution transform. (Received November 12, 1968.)

69T-B30. DOROTHY MAHARAM, University of Rochester, Rochester, New York 14627. Extensions of families of linear maps and of probability measures.

Let L and M be (abstract) linear spaces and $\{L_\alpha | \alpha \in \mathcal{A}\}$ a family of linear subspaces of L. Necessary and sufficient conditions are given on the L_α in order that every consistent family of linear maps $\varphi_\alpha : L_\alpha \to M$ have a linear extension $\varphi : L \to M$. [The conditions are independent of M if $M \neq \{0\}$.] The corresponding question is also settled when L and M are partially ordered (and satisfy certain additional reasonable requirements), the φ_α are all positive, and a positive extension is required. These results are used to give (1) necessary and sufficient conditions for a given consistent family of probability measure μ_α on σ-fields $\mathcal{B}_\alpha$ in a space S to have an extension to a probability measure on a σ-field containing $\bigcup \mathcal{B}_\alpha$, and (2) for every such consistent family of measures on a given family $\{\mathcal{B}_\alpha | \alpha \in \mathcal{A}\}$ of σ-fields to have such an extension. (Received November 20, 1968.)

69T-B31. IRA ROSENBAUM, University of Miami, Coral Gables, Florida, and 18650 Belmont Drive, Miami, Florida 33157. A note on some types of quantified statements of use in analysis.

Letting $N_1 = N_\delta(a -) = (x : -\delta < x - a < 0)$, $N_2 = N_\delta(a +) = (x : 0 < x - a < \delta)$, $N_3 = N_\delta(a') = N_1 \cup N_2$, $N_4 = N_\delta(+\infty) = (x : \delta < x)$, & $N_5 = N_\delta(-\infty) = (x : x < -\delta)$, and defining (1) $\mathrm{Ult}_M P(x)$, read "Ultimately, at M,P(x)," and (2) $\mathrm{Pers}_M P(x)$, read "Persistently, at M,P(x)," respectively, by (1') $(E\delta)(Ax)(x \in N_\delta(M). \to . P(x))$, and (2') $(A\delta)(Ex)(x \in N_\delta(M). \&. P(x))$, where M takes one of the symbols $a-$, $a+$, a', $+\infty$, or $-\infty$, as value, one has $(Ax)P(x) \to \mathrm{Ult}_M P(x)$; $\mathrm{Ult}_M P(x). \to . \mathrm{Pers}_M P(x)$; $\mathrm{Pers}_M P(x). \to . (Ex)P(x)$; $\mathrm{Ult}_M P(x). \leftrightarrow . - \mathrm{Pers}_M - P(x)$; $\mathrm{Ult}_M P(x). \to . - \mathrm{Ult}_M - P(x)$; $- \mathrm{Pers}_M P(x). \to . \mathrm{Pers}_M - P(x)$; $\mathrm{Ult}_M(P(x) \& Q(x)) : \leftrightarrow : \mathrm{Ult}_M P(x). \& \mathrm{Ult}_M Q(x)$; $\mathrm{Ult}_M(P(x) \& Q(x)) : \to : \mathrm{Ult}_M P(x). \& \mathrm{Pers}_M Q(x)$; $\mathrm{Ult}_M(P(x) \& Q(x)). \to : \mathrm{Pers}_M P(x). \& \mathrm{Ult}_M Q(x)$; $\mathrm{Pers}_M(P(x) \vee Q(x)) : \leftrightarrow : \mathrm{Pers}_M P(x). \vee. \mathrm{Pers}_M Q(x)$; $\mathrm{Ult}_M(P(x) \to Q(x)) : \to : \mathrm{Ult}_M P(x). \to \mathrm{Ult}_M Q(x)$; and, for finite a, $\mathrm{Ult}_a P(x). \to . \mathrm{Ult}_{a-} P(x)$; $\mathrm{Pers}_{a-} P(x). \to . \mathrm{Pers}_a P(x)$; $\mathrm{Ult}_{a+} P(x). \& \mathrm{Ult}_{a-} P(x) : \leftrightarrow : \mathrm{Ult}_a P(x)$; etc. For M an endpoint of an interval, or of adjacent intervals, of definition of f(x), and L finite, one can define: $\mathrm{Lim\ sup}_M f(x) = L : \leftrightarrow : (A\gamma)(\mathrm{Ult}_M(f(x) - L. < \gamma) \& \mathrm{Pers}_M(-\gamma < f(x) - L))$, and $\mathrm{Lim\ inf}_M f(x) = L : \leftrightarrow : (A\gamma)(\mathrm{Pers}_M(f(x) - L. < \gamma) \& \mathrm{Ult}_M(-\gamma < f(x) - L))$; while, for $L = \pm\infty$, one can write: $\mathrm{Lim\ sup}_M f(x) = +\infty : \leftrightarrow : (A\gamma)\mathrm{Pers}_M(f(x) > \gamma)$; $\mathrm{Lim\ sup}_M f(x) = -\infty : \leftrightarrow : (A\gamma)\mathrm{Ult}_M(f(x) < -\gamma)$; $\mathrm{Lim\ inf}_M f(x) = +\infty : \leftrightarrow : (A\gamma)\mathrm{Ult}_M(f(x) > \gamma)$; and $\mathrm{Lim\ inf}_M f(x) = -\infty : \leftrightarrow : (A\gamma)\mathrm{Pers}_M(f(x) < -\gamma)$; theorems follow easily. (Received November 18, 1968.)

69T-B32. BENJAMIN G. KLEIN, Courant Institute, New York, New York 10011. Homomorphisms of symbolic dynamical systems.

Let (X,T) be the shift dynamical system on n ($n > 1$) symbols. Let M be a nonperiodic Sturmian minimal set in X or a substitution minimal set in X. Then it is proved that (M,T) is strictly ergodic. Moreover, in each case, the method of proof allows an explicit determination of the unique invariant measure on M. This determination leads, in each case, to a proof that (M,T) has topological entropy zero and also to a proof of Theorem 1. There exists no continuous homomorphism from a substitution minimal set to a nonperiodic Sturmian minimal set. The notion of total minimality leads to a proof of Theorem 2. There exists no continuous homomorphism from a nonperiodic Sturmian minimal set to a substitution minimal set which is not a fixed point of T. (Received November 20, 1968.)

69T-B33. WILLIAM STENGER, Georgetown University, Washington, D. C. 20007. Perturbations and intermediate problems for eigenvalues.

In the method of intermediate problems and the new maximum-minimum theory a significant role is played by operators of the form (1) Q_nAQ_n. A is a selfadjoint operator with domain D in Hilbert space H and has an initial discrete spectrum; Q_n is a projector onto a subspace having co-dimension n. Kuroda [Sci. Papers Colloq. Gen. Ed. Univ. of Tokyo 2 (1961), 1-12] replaces (1) by (2) $A - Q_nAP_n - P_nA$ where $P_n = I - Q_n$. It has been repeatedly asserted, see for instance [T. Kato, Perturbation theory of linear operators, Springer, 1966] that (1) can be reduced to (2) and therefore that (1) yields a perturbation of A. However, it is shown that (1) cannot always be put in the form (2). In fact, the introduction of Q_nAP_n requires that $P_nH \subset D$. Otherwise (2) would not be selfadjoint. On the other hand, (1) is always selfadjoint, see [Bull. Amer. Math. Soc. 74 (1968), 369-372]. When A is bounded (1) and (2) are equal but (2) does not yield the new maximum-minimum theory. Moreover, for the computation of eigenvalues (1) requires an $n \times n$ matrix while (2) requires a $2n \times 2n$ matrix. No numerical applications of (2) are known. (Received December 9, 1968.)

69T-B34. SHAIR AHMAD, Oklahoma State University, Stillwater, Oklahoma 74075, and ALAN C. LAZER, Case Western Reserve University, Cleveland, Ohio 44106. Oscillatory solutions of a third order linear differential equation.

Let p, q and r be continuous and nonnegative for $a \leqq t < \infty$. A nontrivial solution of (L) $y''' = py'' + qy' + ry$ is oscillatory if its set of zeros is not bounded above. Theorem. The following conditions are equivalent: (1) (L) has an oscillatory solution. (2) If w is a nontrivial nonoscillatory solution of (L) there exists c such that, for $t \geqq c$, $w(t)w'(t)w''(t) \neq 0$ and $\operatorname{sgn} w(t) = \operatorname{sgn} w'(t) = \operatorname{sgn} w''(t)$. Theorem. If there exists one oscillatory of (L) then there exists two linearly independent solutions u and v of (L) such that any nontrivial linear combination of u and v is oscillatory. Theorem. If $p \equiv 0$, $2r - q' \geqq 0$, (L) has an oscillatory solution, and u and v are as above, then a solution is oscillatory iff it is a nontrivial linear combination of u and v. (Received November 26, 1968.)

69T-B35. ADI BEN-ISRAEL, Northwestern University, Evanston, Illinois 60201. A unified theory for linear equations and inequalities, real or complex.

Notations. For any set $S \subset C^n$, $S^* = \{y \in C^n : \mathrm{Re}(y,S) \geqq 0\}$. For any $A \in C^{m\times n}$, $b \in C^m$ and a closed convex cone $S \subset C^n$ the system (*) $Ax = b$, $x \in S$ is consistent [asymptotically consistent] if $\{x: Ax = b, x \in S\} \neq \emptyset$ [$\exists \{x_k\} \subset S \ni \lim_{k\to\infty} Ax_k = b$]. Theorem. Let $A \in C^{m\times n}$, $b \in C^m$, S a closed convex cone in C^n. Then [(*) is asymptotically consistent] iff [$A^H y \in S^* \Rightarrow \mathrm{Re}(b,y) \geqq 0$] iff [$b \in R(A)$, $A^+ b \in \mathrm{cl}(N(A) + S)$]. Theorem. Let $A \in C^{m\times n}$, $b \in C^m$, S a polyhedral cone in C^n. Then [(*) is consistent] iff [$A^H y \in S^* \Rightarrow \mathrm{Re}(b,y) \geqq 0$]. Corollary. $Ax = b$ is consistent iff $A^H y = 0 \Rightarrow (b,y) = 0$. Corollary (Levinson). Let $\alpha \in R^n$ satisfy $0 \leqq \alpha \leqq \pi/2$. Then [$Ax = b$, $|\arg x| \leqq \alpha$ is consistent] iff [$|\arg A^H y| \leqq \pi/2 - \alpha \Rightarrow \mathrm{Re}(b,y) \geqq 0$]. Corollary (Farkas). Let $A \in R^{m\times n}$, $b \in R^m$. Then [$Ax = b$, $x \geqq 0$ is consistent] iff [$A^T y \geqq 0 \Rightarrow (b,y) \geqq 0$]. (Received November 27, 1968.)

69T-B36. STERLING K. BERBERIAN, University of Texas, Austin, Texas 78712. Some conditions on an operator implying normality.

For terminology, see P. R. Halmos, A Hilbert space problem book [Van Nostrand, Princeton, N. J., 1967]. An operator is normal if and only if every quadratic polynomial in the operator and its adjoint is convexoid [normaloid]. An operator whose restriction to every invariant subspace is convexoid, and whose spectrum has only finitely many limit points, is normal. An operator on a finite-dimensional Hilbert space is normal if and only if its restriction to every reducing subspace is spectraloid [convexoid, normaloid]. (Received December 5, 1968.)

69T-B37. JAMES D. BUCKHOLTZ, University of Kentucky, Lexington, Kentucky 40506. Successive derivatives of entire functions.

For $n = 0,1,2,\ldots$ and $0 \leqq u < \infty$, let $T_n(u) = \max \sum_{m=n+1}^{\infty} u^m \cdot |\sum_{k=0}^{n} z_k^{m-k} G_k(0; z_0,\ldots,z_{k-1})/(m-k)!|$, where G_k denotes the kth Gontcharoff polynomial, and the maximum is taken over all sequences $\{z_k\}_{k=0}^{n}$ which satisfy $|z_k| = 1$, $0 \leqq k \leqq n$. Let W denote the least positive number u such that $T_n(u) \geqq 1$, $n = 0,1,2,\ldots$. Theorem 1. There is an entire function F of exponential type W such that F and each of its derivatives have a zero on the circle $|z| = 1$. Furthermore, if f is an entire function of exponential type less than W, and f and each of its derivatives have a zero in the disc $|z| \leqq 1$, then $f \equiv 0$. Theorem 2. If f is an entire function of exponential type τ $(0 < \tau < \infty)$ and $\epsilon > 0$, then infinitely many derivatives of f are univalent in the disc $|z| < W/(\tau + \epsilon)$. In view of the function F of Theorem 1, the constant W is best possible here. (Received December 2, 1968.)

69T-B38. BRUCE L. CHALMERS, University of California, Riverside, California 92502. Linear transformations in Hilbert spaces with kernel function.

For $i = 1,2$, let $H_iK_i(E_i)$ be a Hilbert space of functions (defined on E_i) with inner product denoted by $(\ ,\ )_i$ and kernel function $K_i(x,y)$. In the sequel, $T : A \to B$ will denote a bounded linear operator from A into B. Theorem 1. Let $T : H_1K_1(E_1) \to H_2K_2(E_2)$. Then $(Tf)(z) = (f(x), T_y(K_1(y,x))_z)_1$. (Here the y subscript emphasizes that T is operating on $K_1(y,x)$ in $H_1K_1(E_1)$ as a function of y, and the z subscript indicates that for each fixed x, the function $T_y(K_1(y,x))$ in $H_2K_2(E_2)$ is evaluated at $z \in E_2$.) Theorem 2. Let $T : H_1K_1(E_1) \to H_2K_2(E_2)$, where the range of T is dense in $H_2K_2(E_2)$. Then $K_2(w,z) = T_x^{*-1}(\overline{T_y(K_1(y,x))_z})_w$ where T* denotes the adjoint of T. As a special case of Theorem 1, we obtain Theorem 3. If $\mathcal{L} \in HK^*$ (the dual of HK), then the Riesz representation of $\mathcal{L}$ is the function $L = \overline{\mathcal{L}_y K(y,x)}$. Theorem 4. If $\mathcal{L}^m \in HK^*$, $m = 1,\dots, N$, are linearly independent then $\mu = \inf \|f\|$ under the side conditions $\mathcal{L}^m(f) = a_m$, $m = 1,\dots, N$, has the unique solution $f = -D_a^{\mathcal{L}}/D$ with $\mu^2 = -D_a^{\bar{a}}/D$. (Here $D = |Q|$, where Q is $N \times N$ matrix $[(L_k, L_m)]$, where $L_p = \overline{\mathcal{L}_y^p K(y,x)}$, $D_w^v = |Q_w^v|$, where Q_w^v is $(N + 1) \times (N + 1)$ matrix with Q as lower right-hand block and $\vec{ov}$ as first row and $(\vec{ow})^T$ as first column, $\mathcal{L} = (L_1,\dots, L_n)$, and $a = (a_1,\dots, a_n)$.) (Received December 2, 1968.)

69T-B39. BYRON L. EHLE, University of Waterloo, Waterloo, Ontario, Canada. On Padé approximations to the exponential.

Let $P_{mn}(z)$ be the unique rational Padé approximation to the exponential function exp(z) whose numerator is a polynomial of degree n and whose denominator is a polynomial of degree m. It has been shown by Birkhoff and Varga (J. Math. Phys. 44 (1965), 1-23) that the diagonal Padé approximations, $P_{nn}(z)$, have the property that for all n, $|P_{nn}(z)| \leq 1$ for all z such that $R(z) \leq 0$. Clearly this relation does not hold for $n > m$. The following two theorems have been proved dealing with the below diagonal Padé approximations. Theorem. For all n, $|P_{n+1,n}(z)| \leq 1$ and $|P_{n+2,n}(z)| \leq 1$ for all z such that $R(z) \leq 0$. Theorem. For all n, $|P_{n+k,n}(z)|$ is not bounded by 1 for $k \geq 3$ for all z such that $R(z) \leq 0$. (Received December 2, 1968.)

69T-B40. SAMUEL ZAIDMAN, University of Montreal, Quebec, Montreal, Canada. A property of some homogeneous symbols.

Let $a(x,\xi)$ be a complex-valued function defined for $x \in R^n$ and $\xi \in R^n - \{0\}$, such that $a(x,t\xi) = a(x,\xi)$ for $t > 0$, $a(\infty, \xi) = \lim_{(x)\to\infty} a(x, \xi)$ exists for each $\xi \in R^n - \{0\}$ and $a(\infty,\xi) \in C^1(R^n - \{0\})$. The function $a'(x, \xi) = a(x,\xi) - a(\infty, \xi)$ is C^∞ with respect to x and C^1 with respect to ξ. The following estimates are true: $(1 + |x|^2)^p |D_x^\alpha D_\xi^\beta a'(x, \xi)| \leq C_p$, α, β, for $x \in R^n$, $|\xi| = 1$, each multi-index α, each multi-index $\beta = (\beta_1, \beta_2,\dots, \beta_n)$ such that $\beta_1 + \dots + \beta_n = 1$, each natural p. Then if $\tilde{a}'(\lambda, \xi) = (2\pi)^{-n/2} \int e^{-ix\cdot\lambda} a'(x,\xi)dx$, for $\xi \neq 0$, $\lambda \in R^n$, we have $(1 + |\lambda|^2)^p |\tilde{a}'(\lambda,\xi) - \tilde{a}'(\lambda,\eta)| \leq C_p|\xi - \eta|(|\xi| + |\eta|)^{-1}$, $\xi, \eta \in R^n - \{0\}$, $\lambda \in R^n$, $p = 1,2,\dots$.
(Received November 12, 1968.)

69T-B41. PAUL L. ROSENTHAL, Stanford University, Stanford, California 94305. On the analytic continuation of the stream function of a compressible fluid given by its data on the transonic line.

S. Bergman in Amer. J. Math. 70 (1968), 856-891, and 74 (1952), 444-474, shows that a solution ψ (in the pseudo-logarithmic plane) to the partial differential equation $\psi_{xx} + \iota(x)\psi_{yy} = 0$, where $\iota(x)$ is analytic and $\iota(x) > 0$ for $x < 0$, $\iota(x) < 0$, $x > 0$, can be represented in the subdomain $W = \{3^{1/2} |x| < y, x < 0, y > 0\}$ of the subsonic region as an integral of an analytic function f(z) of one complex variable $z = (x + iy)(1 - t^2)/2$. Using the theory of integral operators for equations of mixed type and a result of Schiffer and Siciak in Studies in Mathematical Analysis and Related

Topics, Stanford University Press, 1962, pp. 341-359, the author gives necessary and sufficient conditions in terms of $\lim_{x\to 0^-} \psi(x,y)$ and $\lim_{x\to 0^-} (-\lambda)^{1/3}\, \partial\psi/\partial x$ in order that ψ can be analytically continued (1) in the wedge domain W, (2) in any domain $B \subset W$, satisfying certain conditions. It is assumed that the above limits are analytic (real) functions of y on a segment $0 < y \leq y_0$ of the transonic line $x = 0$. (Received December 5, 1968.)

69T-B42. V. I. GAVRILOV, Indian Institute of Technology, Powai, Bombay-76, India. The distribution of the values of meromorphic functions in the unit disk.

In the paper (V. I. Gavrilov, Mat. Sb. 113 (71) (1966), 386-404) the distribution of the values of meromorphic (holomorphic) functions in the unit disk D is studied in terms of P_h-sequences (ρ_h-sequences). It is shown that any ρ_h-sequence for a holomorphic function f(z) is a P_h-sequence, and, conversely, any P_h-sequence for f(z) contains a subsequence which is a ρ_h-sequence. These results are, in turn, the generalizations of those obtained by the author in a previous paper for P-sequences and ρ-sequences. Very recently, P. Gauthier (Nagoya Math. J. 32 (1968), 277-282) has extended the concept of a ρ-sequence so as to be applicable to meromorphic as well as to holomorphic functions in D and has proved that a sequence $\{z_n\}$ of points in D is a ρ-sequence for a meromorphic function f(z) if and only if $\{z_n\}$ is a P-sequence for f(z). We extend the concept of a ρ_h-sequence and show that Gauthier's theorem remains valid for ρ_h-sequences (in the new sense) and P_h-sequences. (Received December 2, 1968.) (Author introduced by Professor Padam C. Jain.)

69T-B43. HANS F. GŰNZLER, University of Florida, Gainesville, Florida 32601. Weak and strong almost-periodicity.

For a function f defined on the reals with values in a Banach space B there are at least two possible definitions of almost-periodicity: The strong almost-periodicity uses Bohr's definition, replacing the absolute value by the norm; f is weakly a.p. means for each continuous linear functional h on B, the composition $h \circ f$ is a.p. There are two situations when the two concepts coincide: First, when the range space is l^1; secondly, when f is of the form $f(t) = T(t)x$, T(f) a one-parameter equicontinuous group of transformations in B. For the proof one has to show that a mean value for a.p. functions exists and the approximation theorem is valid if the range space is only sequentially complete instead of topologically complete. Generalizations to functions defined on arbitrary semigroups are possible, the second result can be used in the study of a.p. solutions of partial differential equations. (Received November 15, 1968.)

69T-B44. DOROTHY BROWNE SHAFFER, Fairfield University, Fairfield, Connecticut 06433. On the convexity of lemniscates.

Let $f(z) = \Pi_{\nu=1}^n (z - z_\nu)$, denote by $E : |f(z)| \leq 1$ and by L_1 the lemniscate $|f(z)| = 1$. Theorem 1. Let the poles z_ν be inscribed in the disc $|z| \leq a$. Assume $n^{-1}\Sigma_{\nu=1}^n z_\nu = 0$. Then L_1 is convex if $a \leq 1/[3]^{1/2} \approx .578$. The theorem is established by means of the author's previous result (J. Analyse Math. 17, 59-70) and a result of Pommerenke (Michigan Math. J. (1961), 99) which implies that the disc $|z| \leq (1 - a^2)^{1/2} \subset E$. Theorem 2. Let the poles z_ν be inscribed in the disc $|z| \leq a$

and let $z_0 = n^{-1}\sum_{\nu=1}^{n} z_\nu$. Then L_1 is convex if $a \leqq \{[2|z_0|^2 + 3]^{1/2} - [2]^{1/2}|z_0|\}/3$. Theorem 3. Let all the z_ν be real and $\in [-a,a]$, then L_1 is convex if $a \leqq 1/[2]^{1/2}$. (Received December 5, 1968.)

69T-B45. LEONARD STERNBACH, Ohio State University, Columbus, Ohio. On quasi-reflexive spaces.

We say a Banach space X is quasi-reflexive of order n if the natural embedding of X into its second conjugate has codimension n. Theorem 1. If X is quasi-reflexive of order n and $0 \leqq k \leqq n$, then there is a separable subspace of X which is quasi-reflexive of order k. Theorem 2. If in Theorem 1 X has a boundedly complete basis (x_i), then there is a block basic sequence (z_i) of $[x_i]$ such that the order of $[z_i]$ is k. (z_i a block basic sequence of (x_i) means $z_i = \sum_{k;p_i+1}^{p_{i+1}} a_k x_k$ for some a_k and for $1 = p_1 < p_2 < \dots$.) (Received December 2, 1968.)

69T-B46. WITHDRAWN.

69T-B47. PETER D. MORRIS, Pennsylvania State University, University Park, Pennsylvania 16802. Extreme extension operators.

Let X be a compact metric space, let A be a closed subset of X, and let $\mathscr{E}$ be the set of positive, norm-preserving, linear extension operators from $C_R(A)$ into $C_R(X)$. Then $\mathscr{E}$ is convex and its extreme points are precisely its multiplicative elements. The proof of the latter assertion uses a result of R. R. Phelps (Trans. Amer. Math. Soc. 108 (1963), 265-274). Theorem. $\mathscr{E}$ is the closed (in the weak-operator topology) convex hull of its extreme points iff $X \setminus A$ is totally disconnected. Corollary. Let Y and Z be compact metric spaces and let K_0 denote the set of all positive operators of norm $\leqq 1$ from $C_R(Y)$ into $C_R(Z)$. Then K_0 is the closed (in the weak-operator topology) convex hull of its extreme points iff Z is totally disconnected. Remark. According to a result of Phelps (loc. cit.), the extreme points of K_0 are precisely its multiplicative elements. (Received December 16, 1968.)

69T-B48. AMRAM MEIR and AMBIKESHWAR SHARMA, University of Alberta, Edmonton 7, Alberta, Canada. On Illyeff's conjecture.

The conjecture of Illyeff states that if all zeros of a polynomial P(z) lie in the closed unit disk and if $P(\zeta) = 0$, then at least one zero of $P'(z)$ lies in $|z - \zeta| \leqq 1$. We obtain a result which implies the above conjecture for quintics. Among other results we prove the following: If $P(z) = (z - \zeta)^k, \prod_1^{n-k}(z - z_j)$, $|z_j| \leqq 1$, $|\zeta| = 1$, $z_j \neq \zeta$, then at least one zero of $P^{(\nu)}(z)$ lies in $|z - k\zeta/(\nu+1)| \leqq 1 - k/(\nu+1)$. (Received December 9, 1968.)

69T-B49. C. J. EVERETT, Los Alamos Scientific Laboratory, Los Alamos, New Mexico. A note on double sequences.

A result of F. London (Ueber Doppelfolgen ..., Math. Ann. 53 (1900), 338,340) states that, if the row and column limits of a double sequence $D^m n$ exist, and one is uniform, so is the other, and both iterated limits and the double limit exist, and are equal. As an immediate corollary: If

$D^m = \sum_0^\infty d_k^m$ are series with partial sums D_n^m uniformly convergent to D^m, and if $\lim d_k^m = d_k$, then $\lim D^m = \sum_0^\infty d_k$. As classical examples one obtains $\lim (1 + z/m)^m = e^z$ and $\Pi_1^\infty (1 - z^2/m^2) = \sin \pi z/\pi z$. The latter requires a generalization of the relation $\zeta(2) = \pi^2/6$ in the form $\lim_m \sigma_k(1/1^2, \ldots, 1/m^2) = \pi^{2k}/(2k+1)!$, where σ_k is the kth elementary symmetric function. A further example is the curious series $2 = \log^{-1} 2 + 1/2 + \sum_0^\infty B_k(\log 2)^{k-1}/k!$ involving the Bernoulli numbers. This follows from the corollary and results cited in Abstract 68T-H2, these *Notices* 15 (1968), 813. (Received December 27, 1968.)

69T-B50. JACOB BURBEA, Stanford University, Stanford, California 94305. On the kernel function in domains with Bergman-Šilov boundary surface.

In [Math. Z. 63 (1955), 173-194] analytic polyhedra $\mathfrak{M}$ have been introduced; they possess the Bergman-Šilov boundary surface $B(\mathfrak{M})$. Let $\mathcal{L}^2H(B(\mathfrak{M}))$ be the separable Hilbert space of holomorphic functions f of two complex variables (z_1, z_2) regular in $\mathfrak{M}$ and possessing the boundary values on $B(\mathfrak{M})$ almost everywhere, such that $\|f\|^2 = \iint_{B(\mathfrak{M})} |f(z_1, z_2)|^2 d\Omega < \infty$. By introducing the scalar product $(f,g) = \iint_{B(\mathfrak{M})} f\bar{g}\, d\Omega$ and using the Bergman-Weil formula, one can show that $\mathcal{L}^2H(B(\mathfrak{M}))$ is a separable Hilbert space, and by exploiting the representation theorem one obtains the kernel function $\hat{K}_{\mathfrak{M}}(z,\bar{t})$. It holds $\hat{K}_{\mathfrak{M}}(z,\bar{t}) = \sum_{\nu=1}^\infty \varphi_\nu(z)\overline{\varphi_\nu(t)}$, where $\{\varphi_\nu\}_{\nu=1}^\infty$ is a complete orthonormal system. $\hat{K}_{\mathfrak{M}}(z,\bar{t})$ can be considered as a generalization of the Szegő kernel in C^1. Let $K_{\mathfrak{M}}(z,t)$ be the Bergman kernel function of $\mathcal{L}^2H(\mathfrak{M})$. Then $K_{\mathfrak{M}}(z,\bar{t})/\hat{K}_{\mathfrak{M}}^2(z,\bar{t})$ is invariant under a PCT (pseudo-conformal transformation). This fact yields the Theorem. Let $W : \mathfrak{M} \to \mathfrak{M}^*$ be a PCT and let $\mathfrak{M} \subset \mathfrak{M}^*$, then $\hat{K}_{\mathfrak{M}}(z,\bar{z}) \geq \hat{K}_{\mathfrak{M}^*}(z,\bar{z})$. Proceeding in the usual way, the author introduces certain Hermitian metrics by using $\hat{K}_{\mathfrak{M}}(z,\bar{z})$. These metrics are invariant with respect to a PCT. The invariants connected with one of these metrics are represented in terms of solutions of certain minimal problems. Using some of these metrics, an analogue of the Schwarz-Pick lemma is obtained. (Received December 12, 1968.)

Applied Mathematics

69T-C4. JOHN DECICCO and ROBERT DARTSCH, Illinois Institute of Technology, Chicago, Illinois 60616. Elements of a general physical system S_k in Euclidean space E_3.

A general physical system S_k of trajectories C in a Euclidean space E_3 is that obtained by certain constrained motions and by the influence of a general force field F, which depends on the position of the point, the velocity vector at the point, and the time t. Such a force field is assumed to be not a scalar multiple of the velocity vector. The parametric and explicit differential equations of such a physical system S_k show that the total number of such paths C may be either ∞^6 or ∞^5. Under suitable differentiability conditions, a general physical system S_k is composed of all the integral solutions C of three ordinary differential equations, the first one expresses t' in terms of $y'^{1/2}$, the next one defines z" as a linear multiple of y", and the remaining one expresses y"' as a linear combination of y", y''^2, and $y''^{5/2}$. The corresponding coefficients involve t, x, y, z, y', z', and the speed v. This system is equivalent to one of the type (G) if and only if it is independent of t and t', in which case, there are ∞^5 trajectories C. (Received November 12, 1968.)

69T-C5. DALE W. LICK, University of Tennessee, Knoxville, Tennessee 37916. A divergence theorem for a nonlinear Dirichlet problem.

Consider the Dirichlet problem (*) $\Delta u = f(u)$, $u \geqq 0$, in G, $u = \varphi$, on Γ, where Δ is the Laplacian, f may be nonlinear, φ is a continuous function, and G is a bounded domain in R^n with boundary Γ. This note establishes a condition on mesh size under which a common successive approximation technique fails to converge to the solution of a standard finite difference analogue of (*). (Received November 5, 1968.)

69T-C6. WITHDRAWN.

69T-C7. DALLAS J. HOWE and RAM P. MANOHAR, University of Saskatchewan, Saskatoon, Saskatchewan, Canada. Stability of a generalized corrector formula.

In 1962 Crane and Lambert developed some generalized corrector formulas of order four for solving ordinary differential equations numerically. In this paper a modification of their procedure is introduced. A discussion of corrector formulas of order four together with their range of stability and their truncation errors is given. This is followed by a study of corrector formulas of order four which are relatively stable. Finally, a corrector formula is obtained which has maximum range of absolute stability and minimum possible truncation error. Usefulness of this relatively stable formula for solving ordinary differential equations when the solutions decay exponentially is demonstrated by a numerical example. (Received November 11, 1968.)

69T-C8. MICHAEL A. ARBIB, Stanford University, Stanford, California 94305. A characterization theorem for multilinear systems.

Let $U_1, U_2, \ldots, U_r$, and Y be vector spaces, and let U* be the set of finite sequences with elements in $U = U_1 \times U_2 \times \ldots \times U_r$. We say that a function $f : U^* \to Y$ is multilinear if $f(0\theta) = f(\theta)$ for all θ in U* and if wherever we fix input sequences of length n in all but the jth argument, then the resultant function $U_j^n \to Y$ is always linear. Then we show that a function f is multilinear iff it is the input-output function of a system of r layers, with $\binom{r}{j}$ linear machines in the jth layer, one for each j-element subset of $\{1, \ldots, r\}$, where each machine only receives input from inputs and machines corresponding to subsets of its subset, combined at most via multilinear memoryless maps, with the output of the network being the output of the unique machine in layer r. (Received November 25, 1968.)

69T-C9. EINAR HILLE, University of New Mexico, Albuquerque, New Mexico 87106. Observations on the equations of Emden and of Thomas-Fermi.

Emden's equation (1907?) is $y'' = x^{1-m}y^m$ which gives the Thomas-Fermi equation (1927) for $m = 3/2$. For the latter the following problems are solved: (1) convergence of formal solutions in powers of $x^{1/2}$ with $y(0) = 0$ or 1; (2) existence of solution which becomes infinite as $x \uparrow a < \infty$ with estimate $y(x) < 400a(x - a)^{-4}$; (3) existence of solution with preassigned algebraic branch point; (4) determination of all solutions bounded at ∞ and convergence of series $x^{-3}[144 + \sum_1^{\infty} c_n x^{-n\sigma}]$, $\sigma = [\sqrt{73} - 7]/2$. Determination of all solutions unbounded at $x = 0$ and convergence of the series $x^{-3}[144 + \sum_1^\infty d_n x^{n\tau}]$, $\tau = [\sqrt{73} + 7]/2$. Here $144x^{-3}$ is the well-known singular solution and the Thomas-Fermi solution falls under (4) with a value of c_1 obtainable from the Sommerfeld approximation (1932). The results extend to Emden's equation for $1 < m < 3$ with suitable modifications of series and estimates. There is an elementary singular solution and series expansions of type (4) and (5) with the singular solution as first term and σ, τ as roots of a quadratic equation over Q(m). (Received December 6, 1968.)

69T-C10. WILLIAM F. TRENCH, Drexel Institute of Technology, Philadelphia, Pennsylvania 19104. Continuous smoothing with polynomial weighting.

Consider the filter $g(x) = \int_{-1}^{1} W(y)f(x + y)dy$, where W is the polynomial of lowest degree that satisfies (A) $\int_{-1}^{1} W(y)y^s dy = r!\, \delta_{rs}$ $(0 \leqq s \leqq n)$ and (B) $W^{(j)}(1) = W^{(j)}(-1) = 0$ $(\nu \leqq j \leqq \nu + m - 1)$; r, ν, m and n are fixed integers and $0 \leqq r, \nu \leqq n$. (A) implies that $g(x) = f^{(r)}(x)$ if f is a polynomial of degree $\leqq n$, and if $C(\theta) = \int_{-1}^{1} W(x)e^{-ix\theta}dx$, then $\lim_{\theta\to 0} C(\theta)/\theta^r = (-i)^r$. If $|C(\theta)/\theta^r| < 1$ $(\theta \neq 0)$, the filter is defined to be r-stable. The least square filter whose weighting function w_{rn} is the unique polynomial of degree $\leqq n$ that satisfies (A), is r-stable [W. F. Trench, Bounds on the generating functions of certain smoothing operations, Proc. Amer. Math. Soc. 18 (1967), 200-206]. W can be written as a convex linear combination of $w_{rn}, w_{r,n+2}, \ldots, w_{r,n+2m}$, and is consequently also r-stable. If $\nu = 0$, W is the unique function in $C^{(m)}[-1,1]$ that minimizes $Q = \int [W^{(m)}(x)]^2 dx$ subject to (A) and (B). If $\nu = m$, W is the unique function in $C^{(m)}[-1,1]$ that minimizes Q subject to (A) only; (B), with $\nu = m$, are natural boundary conditions for this variational problem. These two extremal properties are analogous to a similar property of minimum R_m smoothing formulas for discrete data. (Received December 13, 1968.)

69T-C11. REINO W. HAKALA, Oklahoma City University, Oklahoma City, Oklahoma 73106. On a sum of reciprocal powers.

The sum $\sum_{k=2}^{\infty}(-1)^k(k - 1)k^{-n}$ has not been evaluated previously in terms of the Riemann zeta function and is evaluated in this paper. The result is used to find the Mellin transform of $(ax)^n(e^{ax} + 1)^{-2}$ which has not been evaluated before. (Received December 20, 1968.)

69T-C12. H. E. GOLLWITZER, University of Tennessee, Knoxville, Tennessee 37916. The Bihari-Langenhop inequalities for Riemann-Stieltjes integrals.

The Bihari-Langenhop inequalities, so useful in the study of differential equations, are also true for Riemann-Stieltjes integrals. Consider $u(t) \leqq C + \int_a^t \omega(u(s))d\eta(s)$, $a \leqq t \leqq b$, where C is a positive constant, ω, η are nonnegative, continuous and nondecreasing, $\omega(u) > 0$ for $u > 0$. Let $\Omega(v) = \int_{u_0}^{v} \omega(s)^{-1}ds$, $v, u_0 > 0$, $\Omega^{-1}(u)$ is the inverse function of Ω. Theorem (Bihari). $u(t) \leqq \Omega^{-1}(k(t))$, $k(t) = \Omega(C) + \int_a^t d\eta(s)$, provided that k is in the range of Ω. Theorem (Langenhop). If u is positive and $u(t) \geqq u(x) - \int_x^t \omega(u(s))d\eta(s)$, $a \leqq t \leqq b$, then $u(t) \geqq \Omega^{-1}(\Omega(u(a)) - \int_a^t d\eta(s))$ for t suitably restricted. The techniques used are general since inequalities of the form $u(t) \leqq f(t) + g(t)\int_a^t W(u(s))d\tau(s) + h(t)\int_a^t \omega(u(s))d\eta(s)$ can also be studied. The above inequalities prove to be useful in the study of the second order differential equation $y'' + q(t)f(y) = 0$. (Received December 30, 1968.)

Geometry

69T-D7. JAMES A. YORKE, University of Maryland, College Park, Maryland 20742. The curvature of closed curves.

Let $\|x\|$ be the Euclidean norm in R^n. Write d/dt as ($'$). A new shorter proof is given of the following Theorem (proved by Fenchel in R^3 and Borsuk in R^n). Let $x : R \to R^n$ be periodic and assume x is C^2 with $x'(t) \neq 0$ (or at least x is C^1 and $x'(t)/\|x'(t)\|$ is absolutely continuous). Write $u(t) = x'(t)/\|x'(t)\|$. Then $\int_0^p \|du(t)/dt\|dt \geqq 2\pi$. The above result can be used to prove Theorem. Let $U \subset R^n$ and $F : U \to R^n$. Assume that for some $L > 0$, F satisfies $\|F(x_1) - F(x_2)\| \leqq L\|x_1 - x_2\|$ for all $x_1, x_2 \in U$. Let x(t) be a nonconstant periodic solution with period p for the equation $x' = F(x)$. Then $p \geqq 2\pi/L$. Actually "$p \geqq 2\pi/L$" is sharp and if $p = 2\pi/L$, then the orbit of x(t) is a circle. (Received November 15, 1968.)

69T-D8. D. W. CROWE, University of Wisconsin, Madison, Wisconsin 53706. Nearly regular polyhedral graphs with two exceptional faces.

Let G(k,2,n) denote a 3-valent polyhedral graph in which the number of edges of each face is divisible by k, except for exactly 2 faces, a, b, between which there is a simple path of n edges. Grünbaum has proved that G(k,2,0) does not exist. In this note we show (i) G(k,2,1) exists if and only if $k = 4$ and $|a| \equiv |b| \pmod 4$. (ii) G(k,2,2) exists if and only if $k = 3$ and $|a| + |b| \equiv 0 \pmod 3$. (iii) G(k,2,3) exists if and only if $k = 4$ and $|a| + |b| \equiv 0 \pmod 4$ or $k = 5$ and $|a| \equiv |b| \pmod 5$. (iv) G(k,2,4) exists if and only if $k = 4$ or 5 and $|a| + |b| \equiv 0 \pmod k$. (v) If the path joining a and b is restricted to be an alternating path ("geodesic"), then G(3,2,n) exists if and only if $n \equiv 2 \pmod 4$ and $|a| + |b| \equiv 0 \pmod 3$. The proofs of (i) - (iv) use simple reductions to known impossible cases. The proof of (v) is by induction on m, where $n = 4m + t$ ($t = 0,1,2,3$), the proofs of (i) - (iv) constituting proof for $m = 0$. (Received November 21, 1968.) (Author introduced by Professor Joshua Chover.)

69T-D9. ARTHUR G. SPARKS, Clemson University, Clemson, South Carolina 29631. On the intersection of maximal L_n sets.

Let S be a set in E_2. For x and y in S, let $\rho(x,y)$ denote the minimum number of segments that a polygonal line joining x to y and lying is S can have. Definition. For x in S, let K_x^n be defined by $K_x^n = \{y \in S | \rho(x,y) \leqq n\}$. The set $K^n = \{x \in S | K_x^n = S\}$ is called the nth order kernel of S. Definition. The set S is said to be an L_n set if for every x in S, it is true that $K_x^n = S$, i.e. $K^n = S$. In the following definition, lemma, and theorem, S will be a compact simply-connected set in E_2. Definition. Let p,q be in S and let C(p,q) be a polygonal path from p to q in S. Let V [C(p,q)] denote the length of C(p,q). Then C(p,q) is called a minimal 1-path if C(p,q) = [p,q]. Let $k > 1$, then C(p,q) is called a minimal k-path if (i) C(p,q) is a k-path; (ii) $p \notin K_q^{k-1}$; and (iii) if C'(p,q) is any other k-path from p to q in S, then $V[C(p,q)] \leqq V[C'(p,q)]$. Lemma. Suppose x is in S and p,q are in K_x^n. Then there is a minimal k-path $C_k(p,q)$ from p to q in S for some $1 \leqq k \leqq 2n$. Furthermore, $C_k(p,q) \subset K_x^n$. Theorem. Let $\mathcal{L}_n$ be the set of all maximal L_n subsets of S. Then $\bigcap \mathcal{L}_n$ is an L_n set. (Received December 2, 1968.)

69T-D10. A. H. STONE, University of Rochester, Rochester, New York 14627. A countable, connected, locally connected Hausdorff space.

An example of a connected, locally connected Hausdorff space with $\aleph_0$ points is constructed, in answer to a question raised by S. P. Franklin. Moreover, the space can be made to satisfy the Urysohn separation axiom, or even stronger ones--but not, of course, regularity. The method is similar to Urysohn's construction (Math. Ann. 94 (1925), 262-295) of a countable connected Hausdorff space. (However, Urysohn's example, like the simpler ones due to Bing and Brown, is neither locally connected nor "Urysohn".) (Received December 19, 1968.)

69T-D11. RAPHAEL M. ROBINSON, University of California, Berkeley, California 94720. Sets of points in space or on a hypersphere with each nearest to 12 others.

Consider a set of points in space or on a hypersphere with no two points at a distance less than a from each other but each point at the distance a from 12 others. Equivalently, consider a set of nonoverlapping spheres of diameter a with each touching 12 others; the centers of these spheres form the set of points. Conjectures. (1) Fejes Tóth. In space, the points must lie on a family of parallel planes, with the points in each plane forming the vertices of equilateral triangles which tile the plane. (2) On a hypersphere, there must be 120 points at the vertices of a regular polyhedroid. It is shown that both of these conjectures follow from either of the following conjectures concerning nonoverlapping equal spheres in space: (a) Given two touching spheres, there are at most 18 other spheres touching one or the other of them. (b) Given a central sphere and 12 touching spheres, the center of a 13th outer sphere must have a distance from the center of the central sphere at least 1.27 times the diameter. Fejes Tóth made the stronger conjecture that the best bound is $7/3\sqrt{3}$ = 1.347..., and suggested using this to prove (1). (Received December 30, 1968.)

Logic and Foundations

69T-E9. DOV GABBAY, Hebrew University, Jerusalem, Israel. Craig interpolation theorem for modal predicate calculi.

For terminology and notation see [1] E. J. Lemmon - Semantics for modal logics, J. Symbolic Logic, 1966. [2] Kripke - "Semantical analysis of modal logic. II" in Theory of models, Amsterdam, 1965. A modal quantificational model is taken in the sense of [2], except that the elements of K are classical quantificational models. Ultraproducts of such models are defined and compactness theorem holds. Also it is proved that if two appropriate models are elementary equivalent they have isomorphic ultralimits. From this, Robinson's joint consistency theorem and Craig's interpolation theorem are deduced. This is done for each of the systems C2 D2 E2, T(C), T(D), T, E3 S4 S5 B S2 S3 of [1]. And S6, S7, S8, $S2^n$, D3 of [2]. (Received September 3, 1968.) (Author introduced by Professor Azriel Levy.)

69T-E10. WILLIAM E. MARSH, Talladega College, Talladega, Alabama 35160. Definable automorphisms. Preliminary report.

Let A be a model of a theory in a first order language L and let G be the automorphism group of A. $f \in G$ is definable (with parameters $a_1, \ldots, a_n$) if for some formula φ in L, $\varphi(a,b,a_1,\ldots,a_n)$ holds in A iff $b = f(a)$. Note that if $g \in G$ and φ defines f as above, then $\varphi(a,b,g(a_1),\ldots,g(a_n))$ defines $g \circ f \circ g^{-1}$. Also if f and h in G are definable, so are f^{-1} and $f \circ h$. It follows that (i) the definable automorphisms form a normal subgroup of G, and in fact, (ii) if the set of parameter tuples used in defining the automorphisms of some subgroup H or G is closed under G, then H is normal. An example of (ii) is the group of inner automorphisms of a group, where the set of one-tuples a_1 used in the formula $b = a_1 a a_1^{-1}$ is the whole group, and only the one formula is needed. (Received December 9, 1968.)

69T-E11. STEVEN K. THOMASON, Simon Fraser University, and University of California, Berkeley, California 94720. Sublattices of the r. e. degrees.

A sublattice of the r. e. degrees is a set S of r. e. degrees such that any two members of S have a g.l.b. in the upper semilattice of r. e. degrees, and both the g.l.b. and the l.u.b. are again members of S. We prove that there is a sublattice of the r. e. degrees isomorphic to the lattice of all finite sets of natural numbers (and therefore sublattices isomorphic to all finite distributive lattices) and one isomorphic to the modular nondistributive lattice with five elements. The methods of proof are derived from [C. E. M. Yates, A minimal pair of r. e. degrees, J. Symbolic Logic, 1966]. (Received November 12, 1968.)

69T-E12. RONALD B. JENSEN, Seminar for Logic, Lennestrasse 33a, 53 Bonn, West Germany. Nonconstructible Δ^1_3 sets.

Write S(a) to mean: $a \subset \omega$, $a \notin L$ and a is the unique solution of a Π^1_2 predicate. Assume ZF to be consistent. Then it remains so upon adjoining any one of the following assumptions. (1) $V = L[a]$ for an a such that S(a) and every constructible set of integers is recursive in a. (2) $V = L[a]$ for an a such that S(a) and a has minimal degree of constructibility. (3) $V = L[a]$ for an a such that S(a); the lattice of degrees of constructibility has type $\omega + 1$, the top degree being the only one to contain a nonconstructible real; each intermediate degree contains a subset of ω_1. (1) is obtained by modifying Solovay's original construction of a nonconstructible Δ^1_3 set; (3) by iterated destruction of Souslin trees. The set of Cohen conditions used to obtain (2) satisfies the countable chain condition and is constructed in a manner analogous to the author's construction of a Souslin tree in L. (Received December 23, 1968.) (Author introduced by Professor Gisbert Hasenjaeger.)

69T-E13. BIENUENIDO F. NEBRES, Stanford University, Stanford, California 94305. A syntactic characterization of infinitary sentences preserved under unions of models.

Assume all formulas are in negation normal form. Let Σ_1 be the set of formulas with only existential quantifiers, $\forall_n \Sigma_1$ the set of universal closures of Σ_1-formulas with at most n free variables, $\Pi'\Sigma_1$ ($= \Pi'_2$) the closure of Σ_1 under countable conjunctions and universal quantification. $\mathfrak{A}$ is the ω-union (n-union) of submodels $\mathfrak{B}_k \subseteq \mathfrak{A}$ if every finite subset (subset with n elements) of $|\mathfrak{A}|$ is contained in some $|\mathfrak{B}_k|$. In a recent paper, [(ω_1, ω) properties of unions of models, Lecture Notes in Mathematics 72, 265-268] Weinstein conjectures that φ is preserved under ω-unions (n-unions) iff $\varphi \in \Pi'_2$ ($\forall_n \Sigma_1$). These are established as corollaries of the following theorem: Theorem. Let φ, ψ be sentences in $L_{\omega_1 \omega}$. The following are equivalent: (1) There is a $\sigma \in \Pi'_2$ ($\forall_n \Sigma_1$) such that $\models \varphi \models \sigma \models \psi$. (2) Every ω-union (n-union) of models of φ is a model of ψ. (1) $\Rightarrow$ (2) is immediate. The proof of (2) $\Rightarrow$ (1) makes use of techniques developed by Makkai [Preservation theorems for the logic with denumerable conjunctions and disjunctions, J. Symbolic Logic, to appear]. (Received December 6, 1968.) (Author introduced by Professor Solomon Feferman.)

69T-E14. BARUCH GERSHUNI, Frans van Mierisstraat 37 huis, Amsterdam, Nederland.
The null-totalities and the individuum.

We postulate the existence of all three empty totalities: the plural null-class $\emptyset$, the singular null-class ($\emptyset$) or ( ) and the null-set $\{\emptyset\}$ or $\{\ \}$. Here the representations $\emptyset$, ($\emptyset$) and $\{\emptyset\}$ are not true, while the representations ( ) and $\{\ \}$ are. Here we call a representation of a totality true if it preserves the kind and the cardinality of the original totality. The plural null-class has evidently no true representation at all. The null-totalities have the following properties: I. The plural null-class is nothing, while the two other null-totalities are something. II. The three null-totalities are originally all different from each another, but ( ) and $\{\ \}$ may be postulated to be equal. III. (( )) $\neq$ ( ); $\{\{\ \}\} \neq \{\ \}$. But there may be postulated the sign = instead of $\neq$. IV. The difference between the three null-totalities $\emptyset$, ( ) and $\{\ \}$ on the one side and the individuum i, which has originally no element, on the other is that the latter, conceived as an improper plural class, has an element, namely itself, while the former have never an element. (A pair of brackets reckons not as an element.) V. The same is also the difference between the generalized empty totalities ((...( )...)) and $\{\{...\{\ \}...\}\}$, and i. In IV and V the concept of plural class proves itself as a useful tool. (Received December 5, 1968.)

69T-E15. ROBERT J. GAUNTT, University of Maryland, College Park, Maryland 20742.
Axiom of choice for collections of finite sets. Preliminary report.

Let ZF* be the modification of ZF which permits the existence of urelements. Let $\mathcal{C}_n$ mean "every collection of n element sets has a choice function". For $Z = \{n_1, \dots, n_k\}$, let $\mathcal{C}(Z)$ mean $\mathcal{C}_{n_1} \wedge \dots \wedge \mathcal{C}_{n_k}$. Let $M_{n,Z}$ mean "for every decomposition of n into a sum of positive primes, $n = p_1 + \dots + p_r$, there exist nonnegative integers $m_1, \dots, m_r$ such that $m_1 p_1 + \dots + m_r p_r \in Z$". Assuming ZF* is consistent, Mostowski proved that $M_{n,Z}$ is necessary for $\vdash_{ZF*}(\mathcal{C}(Z) \to \mathcal{C}_n)$. Whether it is also sufficient is not known. Let $\mathcal{D}_n$ mean "every denumerable collection of n element sets has a choice function". Let $\mathcal{J}_n$ mean "every well-orderable collection of n element sets has a choice function". Theorem. If ZF* is consistent, then $M_{n,Z}$ is necessary and sufficient for $\vdash_{ZF*}(\mathcal{C}(Z) \to \mathcal{D}_n)$ and also for $\vdash_{ZF*}(\mathcal{C}(Z) \to \mathcal{J}_n)$. (Received December 9, 1968.) (Author introduced by Professor James C. Owings, Jr.)

69T-E16. JOHN GREGORY, University of Maryland, College Park, Maryland 20742, and 3409 Tulane Drive, Apt. 2, Hyattsville, Maryland 20783. On a finiteness condition for infinitary languages.

In J. Symbolic Logic, 33, 357, Kreisel asks whether the "generalized finiteness theorem" holds for all admissible sets. Consider the following weaker "finiteness theorem", abbreviated WFT, for admissible set X: "For all X-Δ_1 collections Γ of infinite finite-quantifier sentences, Γ has a model iff for every X-aid$^+$ set S, $\Gamma \cap S$ has a model". Then (1): For every countable admissible ordinal α, there is an (uncountable) admissible X for which WFT fails and ord (X) = α. Given a ZFC model, there is a "small" Boolean extension which is a Boolean model for (2): Statement (1) with the word "countable" deleted, (3): There are admissible X of arbitrarily large cardinality for which WFT fails and every X-aid$^+$ set is a member of X. (Received December 9, 1968.) (Author introduced by Professor Carol Karp.)

69T-E17. M. MAKKAI, Mathematical Institute of the Hungarian Academy of Sciences, Rèaltanoda u. 13-15, Budapest. Preservation theorems for pseudo-elementary classes.

Let for i = 1,..., 8, R_i be the following relation between structures of a given similarity type τ. AR_iB iff, resp., B is a substructure of A, a homomorphic image of A, an endomorphic image of A, a strong homomorphic image of A, a retract of A, an ϵ-extension of A, a direct factor of A, $B \times B \cong A \times A$. For i = 9,..., 16, let R_i be R_{i-8}^{-1}. For a class K of structures of τ, $C_{R_i}(K) =_{df} \{B : AR_iB \text{ for some } A \in K\}$, $K^{\omega_1} =_{df} \{A \in K : A \text{ is countable}\}$. We give (proper) classes F_i of sentences such that Theorem 1. For i = 1,..., 16, (i) $C_{R_i}(\mathrm{Mod}(\Sigma) \upharpoonright \tau) = \mathrm{Mod}(\Sigma) \upharpoonright \tau$ for any $\Sigma \subset F_i$. (ii) for any $K \in PC_\Delta$, $C_{R_i}(K) = \mathrm{Mod}(\Sigma) \upharpoonright \tau$ for some set $\Sigma \subset F_i$. Call an infinitary formula with prefix of length ω and with matrix of form $M\Sigma$ where Σ is a countable set of finitary formulas a Svenonius-formula. Suppose τ is countable. We give a set S_i of Svenonius-formulas of τ such that Theorem 2. For i = 1,..., 16, (i) $C_{R_i}(\mathrm{Mod}(\Phi')) = \mathrm{Mod}(\Phi')$ if $\Phi' = \Phi$ or Φ' is a finite approximation of Φ, (ii) for any countable $\tau' \supset \tau$, and set T of sentences of τ', $C_{R_i}^{\omega_1}(\mathrm{Mod}_{\tau'}(T) \upharpoonright \tau) = \mathrm{Mod}^{\omega_1}(\Phi)$ for some $\Phi \in S_i$. The proof uses a method of Svenonius. From Theorem 2 and the actual form of members of S_i known preservation theorems (due to Keisler et al.) follow. (Received December 13, 1968.) (Author introduced by Professor R. L. Vaught.)

69T-E18. A. SADE, 354 Cours de la République, Pertuis, Vaucluse, France. Morphismes sur le groupoïde n-aire des fonctions propositionnelles.

L'atome x et son déni x + 1 sont représentés par l'intérieur d'un contour C_x et son extérieur. Toute proposition sur $\mu = x_1, x_2, \ldots$ est la somme de polynômes fondamentaux $\Pi(x_i + a_i)$. Il y a dualité entre la matrice d'une fonction et l'ensemble des régions qu'elle représente. Toute fonction peut s'exprimer au moyen des 16 opérateurs de Lukasiewicz. En construisant l'inverse d'une fonction on peut bâtir autant de tautologies que l'on veut. $f(x_1, \ldots) = g(x_1 + 1, \ldots) + 1$ définit l'isomorphe g de f. L'isomorphisme respecte l'équivalence des formules sur μ et induit sur les 16 foncteurs binaires la permutation P_7 = AK.EJ.CM.DX.BL.OV.I.H.G.F, $\forall \mu$ P_7 est la seule qui respecte l'équivalence des formules. Σ étant le groupoïde n-aire de toutes les fonctions, les ensembles Ω, Δ des foncteurs isomorphes à eux-même ou à leur déni sont 2 sous groupoïdes de

Σ; $\Delta \to \Omega = f \to f + x$, $x \in \mu$; $\Delta(\mu + t) = f(\mu) + (x + t)g(\mu)$; $f, g \in \Delta(\mu)$. Le groupe d'automorphisme de Σ est $\{P_7\}$. Tout automorphisme de la restriction de Σ à tout $f \in \Sigma$ est défini par un système d'équations linéaires sur μ, dont le déterminant n'est pas nul et applique tout polynôme fondamental sur un autre. Il décrit un groupe Γ, et $\Gamma \cong \mathfrak{S}_{\mathcal{B}\mu}$. On a Card $\Delta(\mu + t)$ = Card $\Omega(\mu + t)$ = Card $\Sigma(\mu)$ = Card $\mathcal{B}\mathcal{B}\mu$. (Received December 16, 1968.)

69T-E19. ERIK ELLENTUCK, Rutgers University, New Brunswick, New Jersey. Almost combinatorial functions. II.

Let ZF^0 = Zermelo-Fraenkel set theory with the axiom of choice replaced by the axiom of choice for sets of finite sets, Δ = Dedekind cardinals and ω = integers. Let $\mathfrak{A}$ be an arbitrarily quantified positive first order sentence in functors for + and $\cdot$. Let $f_1, \ldots, f_k$ be function variables and let $\mathfrak{A}_f$ be the universal sentence obtained from $\mathfrak{A}$ by replacing existential quantifiers by the f_i as Skolem functions. Theorem. $ZF^0 \vdash$ ($\mathfrak{A}$ holds in Δ) if and only if $ZF^0 \vdash$ (there exist almost combinatorial $f_1, \ldots, f_k$ such that $\mathfrak{A}_f$ holds in Δ). This theorem extends our earlier results (Abstract 68T-500, these *Notices* 15 (1968), 650) in that we no longer require our set theory to admit urelemente. (Received December 2, 1968.)

69T-E20. JOHN P. HELM, Purdue University, West Lafayette, Indiana 47907. A note on effectively continuous operators. Preliminary report.

These results answer questions raised by Young in an effective operator, continuous but not partial recursive (Proc. Amer. Math. Soc. 19 (1968), 103-108) and use the terminology of that paper. They include a very strightforward proof of the result mentioned in the title of Young's paper, but do not yield his main theorem. Effectively continuous operators which map the class of all r.e. sets into itself are partial recursive. However, there exist noneffective, effectively continuous operators which map the class of all partial recursive functions into itself, as well as effective, effectively continuous operators which are not partial recursive. Both examples map the total recursive functions in their domains into constant functions. (Received December 20, 1968.)

69T-E21. SAHARON SHELAH, Hebrew University of Jerusalem, Jerusalem, Israel. On stable homogeneity.

The theorems here are generalizations of results by Morley in On categoricity in power, Trans. Amer. Math. Soc. 114 (1965), 514, and improvements of the results of Shelah in Abstract 68T-E17, these *Notices* 15 (1968), 930. We shall use the definitions and notations of Keisler and Morley in On the number of homogeneous models of a given power, Israel J. 5 (1967), 73-78. Let M be a model and D = S(M) (the set of all types of finite sequences of elements of M). M is a model of D if S(M) = D. Definition 1. D is stable in λ if D has a λ^+-homogeneous model of power $\geqq \lambda^+$, M, such that $|\{T(M, \bar{a}, C) : C \text{ an element of } M\}| \leqq \lambda$ for every α-termed sequence $\bar{a}$ of elements of M. Theorem 1. If D is stable in λ, then there exists μ, $\varkappa$, such that D is stable in λ_1 iff $\lambda_1 = \varkappa + \sum_{\chi<\mu} (\lambda_1)^{\chi}$. Theorem 2. If D is stable in λ, M a model of D of power $> \lambda$, A a set of elements of M, $|A| \leqq \lambda$, then in M there exists an indiscernible set on A of power $> \lambda$. There are also some theorems about prime λ^+-homogeneous models of D on sets contained in a model of D. (Received December 26, 1968.) (Author introduced by Professor Michael Rabin.)

69T-E22. WALTER TAYLOR, University of Colorado, Boulder, Colorado 80302. Satisfaction of atomic formulae in pure extensions. Preliminary report.

A relational structure $\mathfrak{A}$ will be called elementarily bounded in chromatic number if there is an absolute upper bound, in the elementary equivalence class determined by $\mathfrak{A}$, to all chromatic numbers of relations formed from relations of $\mathfrak{A}$ by conjunction and permutation of variables. Theorem. $\mathfrak{A}$ is elementarily equivalent to a weakly atomic-compact relational structure iff $\mathfrak{A}$ is elementarily bounded in chromatic number. Corollary. $\mathfrak{A}$ is a pure substructure of a closure of $\mathfrak{A}$ iff $\langle \mathfrak{A}, a: a \in A\rangle$ is elementarily bounded in chromatic number. (Received December 31, 1968.)

Statistics and Probability

69T-F3. NATHANIEL A. FRIEDMAN, State University of New York, Albany, New York 12203. On weak convergence.

For terminology, see [1] Friedman, Katz, and Koopmans, J. London Math. Soc. 43 (1968), 186-190. Theorem 8 of [1] holds if it is only assumed that τ does not admit a finite invariant measure $\gamma \sim m$. This implies that weak convergence of $\{m\tau_n\}$ does not imply the existence of a σ-finite invariant measure $\gamma \sim m$. (Received December 5, 1968.)

69T-F4. NATHANIEL A. FRIEDMAN, State University of New York, Albany, New York 12203, and D. S. ORNSTEIN, Stanford University, Stanford, California 94305. On mixing and partial mixing.

Let $(X, \mathcal{A}, m)$ be the unit interval with Lebesgue measure. Let τ be mixing, $f \in L_1$, and let (k_n) be an increasing sequence of positive integers. Define $f_n(x) = (1/n)\sum_{i=1}^{n} f(\tau^{k_i}(x))$ and $E(f) = \int f dm$. Blum and Hansen (Bull. Amer. Math. Soc. 66 (1960), 308-311) proved that f_n converges to E(f) in the mean. We construct an example such that for a.e. x, $f_n(x)$ does not converge pointwise. We also construct a partially mixing transformation τ for each $\alpha \in (0,1)$ such that τ is not partially mixing for $\alpha + \epsilon$, $\epsilon > 0$. Partial mixing is defined in Abstract 68T-H31, these *Notices* 15 (1968), 946. (Received December 5, 1968.)

69T-F5. K. S. MILLER and M. M. ROCHWARGER, Riverside Research Institute, 632 West 125 Street, New York, New York 10027. Spectral moments of time series.

Let $\{x(t)\}$ be a wide-sense stationary mean zero complex Gaussian process with continuous spectral density $W(\lambda)$. Let p(t) be a real-valued weighting function, and suppose observations are made on p(t)x(t). Using samples of the (discrete) Fourier transform of p(t)x(t), estimators for the normalized mean and standard deviation of $W(\lambda)$ are deduced. The mean and variance of these statistics are computed to order N^{-2} where N is the sample size. It is shown that for a wide class of spectral densities and weighting functions of practical importance, the estimators are asymptotically consistent. (Received December 18, 1968.)

Topology

69T-G14. PING-FUN LAM, Wesleyan University, Middletown, Connecticut 06457. Inverses of homomorphisms of dynamical systems.

A dynamical system (X,h) is a compact metric space X acted on by a self-homeomorphism h. Theorem 1. Let φ be a homomorphism of dynamical systems (X,h) onto (Y,g) and $y \in Y$. If y is recurrent {positively recurrent} {negatively recurrent} {completely recurrent}, then $\varphi^{-1}(y)$ contains a point which is recurrent {positively recurrent} {negatively recurrent} {completely recurrent}. Let (X,h) be a dynamical system, d a metric on X. For positive numbers ϵ we define the set $R(X,\epsilon) = \{x \in X \mid$ there exists a periodic point y such that $d(h^i(x), h^i(y)) < \epsilon$; $i = 0,\dots, n(y)-1\}$, where $n(y)$ is the period of y. Theorem 2. Let φ be a homomorphism of expansive dynamical systems (X,h) onto (Y,g) and $y \in Y$, a periodic point. Suppose $R(X,\epsilon) \cap \varphi^{-1}(y) \neq 0$ for every $\epsilon > 0$. Then there exists a periodic point $x \in X$ such that $\varphi(x) = y$. (Received August 19, 1968.)

69T-G15. C. J. MOZZOCHI, Trinity College, Hartford, Connecticut 06106. A correct system of axioms for a symmetric generalized topological group.

Let $\{G, \cdot\}$ be a group with identity e and power set $P(G)$. Let $\mathcal{T}$ be a topology on G. Let $\mathcal{N}$ be an open base at e. Definition. $\{G, \cdot, \mathcal{T}\}$ is a symmetric generalized topological group iff the following axioms are satisfied: (A_1) for every A,B in $P(G)$ and $N \in \mathcal{N}$, if $AM \cap B \neq \emptyset$ for all $M \in \mathcal{N}$, then there exists $b \in B$ and there exists $W \in \mathcal{N}$ such that $bW \subset AN$; (A_2) for every $N \in \mathcal{N}$, $N = N^{-1}$; (A_3) $\{aN \mid N \in \mathcal{N}\}$ is an open base at a. For each $N \in \mathcal{N}$, let $U_N = \{(x,y) \mid x^{-1}y \in N\}$. Let $\mathcal{B} = \{U_N \mid N \in \mathcal{N}\}$. Theorem. $\mathcal{B}$ is a base for a symmetric generalized uniformity $\mathcal{U}$ on X such that $\mathcal{T}(\mathcal{U}) = \mathcal{T}$ (cf. Abstract 658-54, these *Notices* 15 (1968), 733). Remark. It is easily shown that every topological group is (essentially) a symmetric generalized topological group. (Received November 5, 1968.)

69T-G16. ROBERT F. BROWN, University of California, Los Angeles, California 90024. Divisible H-spaces.

Define an H-space to be a pair (X,μ) where X is a space and $\mu : X \times X \to X$ is a map such that, for some $e \in X$, $\mu(x,e) = \mu(e,x) = x$ for all $x \in X$. Given an H-space (X,μ), define $p_k : X \to X$, $k \geqq 2$, by $p_2(x) = \mu(x,x)$ and, in general, $p_k(x) = \mu(x,p_{k-1}(x))$. Call (X,μ) divisible if p_k is onto for all $k \geqq 2$, and call $x \in X$ a kth root of $y \in X$ if $p_k(x) = y$. Theorem. If (X,μ) is an H-space where X is a compact connected manifold (without boundary), then (X,μ) is divisible. Moreover, if β is the dimension of $H^1(X)$ (rational coefficients), then every $x \in X$ has at least k^{β} kth roots for all $k \geqq 2$. Conjecture. If the exterior algebra $H^*(X)$ has λ generators, then every $x \in X$ has at least k^{λ} kth roots. The conjecture is known to be true if (X,μ) is a Lie group (H. Hopf, Comment. Math. Helv. 13 (1940), 119-143). (Received November 12, 1968.)

69T-G17. R. D. ANDERSON, Louisiana State University, Baton Rouge, Louisiana 70803. A characterization of apparent boundaries of the Hilbert cube.

Let s denote any countable infinite product of open intervals and let I^∞ denote the countable infinite product of the closures of those intervals. Let $B(I^\infty) = I^\infty \setminus s$. A set M in I^∞ is an apparent boundary if there exists a homeomorphism of I^∞ onto itself carrying M onto $B(I^\infty)$. A coordinate-free topological characterization of apparent boundaries is given as follows. A closed set K in I^∞ has Property Z if for each nonempty homotopically trivial open set U, $U\setminus K$ is nonempty and homotopically trivial. A set M in I^∞ has the two compacta approximation property (TCAP) if for any $\epsilon > 0$ and any compacta K_1 and K_2 in I^∞ with $K_1 \subset M$ there is a homeomorphism f of $K_1 \cup K_2$ into M such that $f|K_1 = \mathrm{id}$ and $d(f, \mathrm{id}) < \epsilon$. Theorem. A set M in I^∞ is an apparent boundary iff M has the TCAP and M is the countable union of closed sets with Property Z. The proof of the theorem uses considerable previous apparatus (e.g. Theorem II, R. D. Anderson, Strongly negligible sets in Fréchet manifolds, to appear in Bull. Amer. Math. Soc.) and the extension theorem of Barit (Abstract 663-715, these *Notices* 16 (1969), 295). (Received November 8, 1968.)

69T-G18. VINCENT J. MANCUSO, St. John's University, Jamaica, New York 11432, and 161-34 28th Avenue, Flushing, New York. Ascoli theorems for multivalued functions.

The notions of collective continuity and even continuity are extended to include multivalued functions. Then using the point-open and compact-open topologies defined by Smithson (Abstract 67T-248, these *Notices* 14 (1967), 290), we prove two Ascoli theorems for a family $\mathfrak{F}$ of multivalued functions. For example, we prove the Theorem. Let X be a k-space and Y a normal Hausdorff space. Let $\mathcal{C} = \{f \in Y^{mX}$; f is continuous and point-compact$\}$ have the compact-open topology (K-topology). Then a family $\mathfrak{F} \subset \mathcal{C}$ is compact if and only if (a) $\mathfrak{F}$ is K-closed in $\mathcal{C}$, (b) $\mathfrak{F}[x]$ is compact for each $x \in X$ and (c) every closed subset of $\mathfrak{F}$ is collectively continuous. For the known single-valued analogue of this result see Gale (Compact sets of functions and function rings, Proc. Amer. Math. Soc. 43 (1950), 303-308). (Received November 8, 1968.)

69T-G19. JAMES M. BOYTE, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Point-paracompactness. Preliminary report.

Definition. Point-(countably)-paracompact (see J. M. Boyte, Abstract 660-2, these *Notices* 15 (1968), 1007). Correction in definition of infinitely covered sequence (see J. M. Boyte, ibid.). Definition. A sequence $\{x_n\}$ is said to be infinitely covered iff there exists a collection of open sets $\{G_n\}_{n=1}^\infty$ such that $x_n \in G_n$ for each n, $x_n \in G_k$ for $k \leq n$ iff $x_n = x_k$, and $\overline{\bigcup_{n=1}^\infty \{x_n\}} \subset \bigcup_{n=1}^\infty \{G_n\}$. Definition. A space X is absolutely-H-closed iff every open cover of the space has a finite subset $\{G_k\}_{k=1}^N$ such that $X = \overline{\bigcup_{k=1} \{G_k\}}$. Theorem 1. A space X is point-(countably)-paracompact iff every (countable) open cover of X has, for each point $a \in X$, a finite subset whose union contains the closure of some neighborhood of a. Theorem 2. A space is compact iff it is point-paracompact and absolutely-H-closed. Theorem 3. In a Lindelöf space, point-countable-paracompactness is equivalent to paracompactness. Theorem 4. A T_2 space is normal iff every open cover $\{G_\alpha\}_{\alpha \in A}$ of X has, for each closed set $F \subset G_\alpha$, $\alpha \in A$, an open refinement covering X, for which there exists N_F such that N_F intersects only finitely many members of the refinement. (Received November 6, 1968.) (Author introduced by Dr. E. P. Lane.)

69T-G20. TUDOR GANEA, University of Washington, Seattle, Washington 98105. Cotriples and Lusternik-Schnirelmann category. II.

For any B of the homotopy type of a connected CW-complex define a sequence of fibrations $p_k : E_k \to B$, $k \geqq 0$, as follows : E_0 is the space of based paths in B, and p_{k+1} results by converting into a fibre map the extension $E_k \cup CF_k \to B$ of p_k which maps the fibre of p_k to the base-point. A natural cross section $j_k : E_k \to E_kE_k$ may be defined, and (E_k, p_k, j_k) forms a cotriple in the sense of Eilenberg-Moore [Illinois J. Math. 9 (1965), 381-398]. Theorem. For any m, $k \geqq 0$, there is a natural map $\Gamma_{mk} : E_mE_k \to E_kE_mE_k$ such that $p_k \circ \Gamma_{mk} = 1 : E_mE_k \to E_mE_k$ and $E_k(p_m) \circ \Gamma_{mk} \simeq j_k \circ p_m : E_mE_k \to E_kE_k$. Corollary. If cat $B \leqq k$ with cross section $g : B \to E_k$, there is a map $\gamma : E_m \to E_kE_m$ such that $p_k \circ \gamma = 1 : E_m \to E_m$ and $E_k(p_m) \circ \gamma \simeq g \circ p_m : E_m \to E_k$; in particular, cat $E_m \leqq k$ for any $m \geqq 0$. The first part of the next result was first obtained by F. Takens: If cat $B \leqq k$ with cross section $g : B \to E_k$, then the strong Lusternik-Schnirelmann category of B is $\leqq k + 1$ and $(B, h \circ g)$ is a coalgebra over $(E_{k+1}, p_{k+1}, j_{k+1})$, where $h : E_k \to E_{k+1}$ is the natural embedding. The proofs use the remark that cat $E \cup CF \leqq$ cat B in any fibration $F \to E \to B$. (Received November 5, 1968.)

69T-G21. MAX K. AGOSTON, Wesleyan University, Middletown, Connecticut 06457. Isotopies of imbeddings in the conditionally stable range.

A homotopy criterion for when certain imbeddings $f, g : M^n \to W^m$ differ by a diffeomorphism of W^m can be given in the "conditionally stable" range, i.e. when f,g are ι-connected and $m \geqq 2n - 2\iota + 1$. For simplicity, only the application to the case $W^m = S^m$ is considered here. Now, any two imbeddings $f, g : M^n \to S^{n+k}$ are isotopic in $S^{n+k} \times D^q$ for large q. This isotopy induces a well-defined bundle isomorphism $\varphi(f,g)$ from the stable normal bundle of f to the stable normal bundle of g. Let $E(\xi)$ denote the total space of a bundle ξ. Now let M^n be an ι-connected closed oriented manifold, $0 \leqq \iota < [n/2]$. Theorem 1. Assume that there is a map of triads $\psi : (S^{n+k}; S^{n+k} - E(\nu_f), E(\nu_f)) \to (S^{n+k}; S^{n+k} - E(\nu_g), E(\nu_g))$ such that $\psi | E(\nu_f)$ is a bundle map from ν_f to ν_g which suspends to $\varphi(f,g)$. If $k \geqq \max(n - 2\iota + 1, 3)$, then f|(M-point) is isotopic to g|(M-point). If $k \geqq \max(n - 2\iota + 1, (n + 4)/2)$, then f is isotopic to g. Theorem 2. If $k \geqq n - \iota + 1$, then f|(M-point) is isotopic to g|(M-point). If $k \geqq \max(n - \iota + 1, (n + 4)/2)$, then f and g are isotopic. Both theorems are proved using only surgery techniques. Theorem 2 is an extension below the metastable range of a theorem of Haefliger. (Received October 16, 1968.)

69T-G22. DONALD L. PLANK, Case Western Reserve University, Cleveland, Ohio 44106. Closed ι-ideals in Φ-algebras.

Let A be a uniformly closed Φ-algebra in the following (for terminology, see Henriksen and Johnson, Fund. Math. 50 (1961), 73-94). We consider closed ι-ideals in the natural uniform topology of A. Theorem 1. If I is a proper ι-ideal in A, then A/I is a Φ-algebra $\Leftrightarrow$ I is closed. Corollary. If $M \in \mathfrak{M}(A)$, then $M \in \mathcal{R}(A) \Leftrightarrow$ M is closed. We call A normal if every proper closed ι-ideal in A is contained in a closed maximal ι-ideal in A. Theorem 2. If A is normal, then $\mathcal{R}(A)$ is a dense Lindelőf subspace of $\mathfrak{M}(A)$. Theorem 3. A is ι-isomorphic to C(X), for some Lindelőf space X, $\Leftrightarrow$ (i) $a \in A$ is a unit whenever $a \notin M$ for all $M \in \mathcal{R}(A)$, and (ii) A is normal. As a corollary, we get Brainerd's characterization of C(X), for X a Lindelöf P-space (see Brainerd, Canad. J. Math. 11 (1959), 80-86). (Received November 18, 1968.)

69T-G23. D. J. LUTZER, University of Washington, Seattle, Washington 98105. A metrization theorem for linearly orderable spaces.

In a recent paper, C. J. R. Borges considered metrization of spaces having G_δ-diagonal (On metrizability of topological spaces, Canad. J. Math. 20 (1968), 795-804). For linearly orderable spaces (i.e., spaces whose topology can be obtained as the open interval topology of some linear ordering of the given set), the following theorem can be proved. Theorem. A linearly orderable space X is metrizable if and only if X has a G_δ-diagonal. It is not enough to assume only that every closed subset of X is a G_δ. This theorem can be used to show, for example, that the real line with the left half-open interval topology is not linearly orderable. (Received November 20, 1968.)

69T-G24. DAVID E. KULLMAN, University of Kansas, Lawrence, Kansas 66044. Developable spaces and p-spaces. Preliminary report.

A. V. Arkhangel'skiĭ [Russian Math. Surveys. 21 (1966), 115-162] has pointed out that a completely regular developable space is a strict p-space. (In this report, all p-spaces are assumed to be completely regular.) C. R. Borges [Canad. J. Math. 20 (1968), 795-804] has proved that a regular developable space has a $\overline{G}_\delta$-diagonal. Theorem 1. A strict p-space with a $\overline{G}_\delta$-diagonal is developable. Theorem 2. A metacompact p-space with a G_δ-diagonal has a σ-point finite development. Corollary. A completely regular space has a uniform base if and only if it is a metacompact, semimetrizable p-space. (Received November 15, 1968.)

69T-G25. RONALD H. ROSEN, University of Michigan, Ann Arbor, Michigan 48104. Concerning suspension spheres.

Theorem. Let Z be a compact metric space which is 1-connected and has the homology of S^n. Suppose that $Z \times R^1$ is an unbounded $(n + 1)$-manifold. If $n \geqq 4$ and Z contains an n-cell, then the suspension of Z is a topological $(n + 1)$-sphere. (Received December 5, 1968.)

69T-G26. M. A. SUNDARAM SASTRY, University of Cincinnati, Cincinnati, Ohio 45220. Statistical topological structures.

In 1965, F. J. Wagner (Abstract 625-145, these *Notices* 12 (1965), 588), defined statistical topological and statistical metric spaces. In this paper characterisations of such spaces as topogenous structures are obtained. Theorem 1. Let X be a STS with T*-interior. For each $\lambda \in [0,1]$, let δ_λ be a relation on P(X) defined by $A \,\delta_\lambda\, B$ if $A \subset^\lambda B$. Then (i) δ_λ is a perfect topogenous order for each $\lambda \in [0,1]$. (ii) $A \,\delta_1\, B$, $B \neq X$ imply $A = \emptyset$. (iii) $\lambda \leqq \mu$ implies $\delta_\mu \leqq \delta_\lambda$. (iv) $A \,\delta_{T*(\lambda,\mu)}\, B$ implies there exists $C \subset X$ such that $A \,\delta_\lambda\, C \,\delta_\mu\, B$. If $\{\delta_\lambda\}_{\lambda \in [0,1]}$ be a family of relations on P(X) satisfying the above conditions, and if ${}^\lambda A = \{x \in X \mid \{x\} \,\delta_\lambda\, A\}$, then X becomes a STS with T*-interior $(A,\lambda) \to {}^\lambda A$. Theorem 2. Let X be a SMS with a T-metric such that $T(\lambda,\mu) \geqq \min(\lambda,\mu)$. Then there exists a set D =

$\{d_\lambda | \lambda \in (0,1]\}$ of semimetrics on X, and a probability measure P on D, with $P(d_\lambda \in D | \lambda \in \Omega)$ being the Lebesgue measure on Ω, such that $F_{xy}(\alpha) = P(d \in D | d(x,y) < \alpha)$. (Received November 12, 1968.)

69T-G27. ROBION C. KIRBY, LAURENCE C. SIEBENMANN, Institute for Advanced Study, Princeton, New Jersey 08540, and CHARLES T. C. WALL, University of Liverpool, Liverpool, England. The annulus conjecture and triangulation.

Methods of Kirby (Stable homeomorphisms, preprint) show that the following conjectures $C(k,m)$, $m \geqq 5$, (Kirby's modified by Siebenmann) imply, for example, existence of a PL manifold structure on any metrizable open topological m-manifold. $C(k,m)$: Let $h : D^k \times T^n \to W^m$, $k + n = m \geqq 5$, be homeomorphism onto a PL manifold that gives a PL isomorphism of boundaries (D^k = k-disk; T^n = n-torus, the n-fold product of circles). Then for some finite covering $\widetilde{h} : D^k \times T^n \to \widetilde{W}$ of h, $\widetilde{h}|\partial D^k \times T^n$ extends to a PL homeomorphism. The stronger conjecture $\overline{C}(k,m)$ with h merely a homotopy equivalence can be decided by surgery. Wall has proved $\overline{C}(k,m)$ for $k \neq 3$, and disproved $\overline{C}(3,m)$. First Conclusions. (A) From $\overline{C}(0,m)$: Every homeomorphism of R^m, $m \geqq 5$, is stable; hence the annulus conjecture holds in R^m. (B) On a PL manifold, dim $\geqq 5$, without boundary, decomposible with no 3-handles, the PL structure is unique up to small topological isotopies. (C) On microbundles: If $i < m \geqq 5$, $\pi_i(TOP_m, PL_m)$ is 0 for $i \neq 3$ and Z_2 or 0 for $i = 3$. Hence if M is any manifold, for d large, $M \times R^d$ admits a PL structure provided $H^4(M; Z_2) = 0$. (D) Without Wall's result one can triangulate any closed 4-connected manifold. J. L. Shaneson and W. C. Hsiang have a later proof of $\overline{C}(0,m)$. (Received December 10, 1968.)

69T-G28. GREGORY M. KELLY and B. J. DAY, University of New South Wales, Kensington, N. S. W., 2033, Australia. Preservation of topological identification maps.

Theorem. Let $f : X \to Y$ be a continuous surjection, where X, Y are topological spaces. In order that every pull-back of f be an identification map, the following condition is necessary and sufficient: For each $y \in Y$, and each open covering $\{G_\alpha\}$ of $f^{-1}y$, there is a finite set $\{\alpha_1,\dots, \alpha_n\}$ of indices such that $fG_{\alpha_1} \cup \dots \cup fG_{\alpha_n}$ is a neighbourhood of y. Theorem. Let $f: X \to Y$ be a topological identification map. In order that $f \times 1 : X \times Z \to Y \times Z$ be an identification map for all topological spaces Z, the following condition is necessary and sufficient: For each $y \in Y$, each neighbourhood V of y, and each open covering $\{G_\alpha\}$ of $f^{-1}V$, there is a finite set $\{\alpha_1,\dots, \alpha_n\}$ of indices such that $(fG_{\alpha_1} \cup \dots \cup fG_{\alpha_n})^*$ is a neighbourhood of y, where A^* denotes the intersection of all the open sets containing A. (Received December 9, 1968.)

69T-G29. R. F. DICKMAN, JR., and ALAN ZAME, University of Miami, Coral Gables, Florida 33124. Functionally-compact spaces. Preliminary report.

A topological space X is said to be functionally-compact if for every open filter $\mathcal{U}$ in X such that the intersection A of the elements of $\mathcal{U}$ equals the intersection the closures of the elements of $\mathcal{U}$, $\mathcal{U}$ is the neighborhood filter of A. Theorem 1. There exists a noncompact Hausdorff space which is functionally-compact. Theorem 2. A Hausdorff space X is functionally-compact if and only if every continuous function from X into any Hausdorff space is a closed map. Corollary 1. Every functionally-compact Hausdorff space is minimal Hausdorff. Corollary 2. If a Hausdorff space Y is the continuous image of a functionally-compact Hausdorff space, Y is functionally-compact. Remark. The example of Theorem 1 shows that functionally-compact is not productive or closed hereditary; however, functionally-compact is a clopen hereditary property. There exist minimal Hausdorff spaces which are not functionally-compact. (Received December 13, 1968.)

69T-G30. R. C. KIRBY and L. C. SIEBENMANN, Institute for Advanced Study, Princeton, New Jersey 08540. A triangulation theorem.

Theorem I. Let M^m, $m \geqq 5$, be any metrizable topological manifold without boundary. For any stable reduction ρ, to the piecewise-linear microbundle group, of the tangent microbundle $\tau(M)$ of M, there exists a PL (= piecewise-linear manifold) structure Σ on M realizing ρ. Further if U is an open subset of M with PL structure Σ_0 realizing $\rho|U$, and V is open in M with closure $(V) \subset U$, then Σ can be chosen to extend $\Sigma_0|V$. Remark (see joint note with C. T. C. Wall, this issue of these *Notices*). The group $\pi_i(TOP, PL)$ is 0 if $i \neq 3$ and Z_2 or 0 if $i = 3$. Corollary II. Let M^m be a metrizable topological m-manifold. If $m \geqq 6$, or $m = 5$ and M has empty boundary, there is a well-defined obstruction in $H^4(M; \pi_3(TOP, PL))$ to giving M a PL manifold structure. Corollary III. If M^m is a compact topological manifold, then, even if M be nontriangulable, M has the homotopy type of a finite complex. In fact, the compact total space of any normal disc-bundle of M in R^{m+k} (k large) is triangulable. Theorem I can be proved via a chart by chart argument using the methods of Kirby Stable homeomorphisms (Ann. of Math., to appear). By a different argument, R. K. Lashof (Bull. Amer. Math. Soc., to appear) has proved a weaker version of Theorem I for certain compact manifolds M with $\tilde{K}_0(\pi_1 M) = 0$. (Received December 18, 1968.)

69T-G31. WILLIAM S. BOYD, JR., University of Tennessee, Knoxville, Tennessee 37916. Repairing embeddings of 2-spheres in E^3 with monotone maps. Preliminary report.

Theorem. Let S be a wild sphere in E^3 and $h: S \to E^3$ a homeomorphism of S such that h(S) is tame. Then h can be extended to a monotone map f of E^3 onto E^3 such that $f(E^3 - S) = E^3 - f(S)$. The proof was a strengthened version of a theorem of McMillan which says a cell in E^3 is the intersection of cubes with small handles. By using a recent result of Bing the point inverses of the map can be assumed to be 1-complexes. (Received December 24, 1968.)

69T-G32. ROBERT C. BRIGGS, Tennessee Technological University, Cookeville, Tennessee 38501. More theorems on weak and strong cover compactness. Preliminary report.

For the definitions of weak cover compactness (w.c.c.) and strong cover compactness (s.c.c.), see Abstract 650-24, these *Notices* 14 (1967), 922. Theorem 1. There exists a collectionwise normal, s.c.c. Hausdorff space S such that $S \times [0,1]$ is not s.c.c. Theorem 2. If X is a s.c.c. (w.c.c.) topological space and f is an open and closed mapping from X onto a topological space Y, then Y is s.c.c. (w.c.c.). Theorem 3. A first countable Lindelőf Hausdorff space is paracompact if and only if it is s.c.c. (If the continuum hypothesis is true, a first countable, separable, s.c.c. Hausdorff space need not be regular. Example due to C. W. Proctor, Abstract 660-13, these *Notices* 15 (1968),

1011.) Theorem 4. In a first countable, separable, regular T_1 space, w.c.c. is equivalent to point-collectionwise normality. Theorem 5. In a first countable, separable, normal T_1 space, w.c.c. is equivalent to collectionwise normality. (A first countable, collectionwise normal T_1 space need not be w.c.c. Abstract 68T-326, these *Notices* 15 (1968), 396.) Theorem 6. If a first countable, separable, normal T_1 space is s.c.c., it is w.c.c. and collectionwise normal, but not conversely.
(Received December 24, 1968.)

69T-G33. OFELIA T. ALAS, Universidade de Sao Paulo, FFCL, Caixa Postal 8105, Braxil. Topological groups and uniform continuity.

Let G be a Hausdorff nondiscrete group, U be the right uniformity of G, e be the neutre element of G. Definition. G is a K-group if every continuous real-valued function on G is uniformly continuous of (G,U) into R. Let I denote an infinite set such that (1) there is a family, $(M_i)_{i\in I}$, of open neighborhoods of e, such that $\bigcap\{M_i | i \in I\}$ is not a neighborhood of e; (2) for any family, $(S_j)_{j\in J}$, of open neighborhoods of e, the set $\bigcap\{S_j | j \in J\}$ is a neighborhood of e if $|J| < |I|$. Theorem. If G is a K-group, then, for any open neighborhood V of e, there is a locally finite open covering of G, with cardinality less than $|I|$, which refines the covering $\{Vx | x \in G\}$. Corollary. If G is a K-group and I is countable, then G is pseudocompact. Counterexample. Let M be the set of the ordinal numbers less than the first uncountable ordinal and $\{0,1\}$ be the group modulus 2. Put $G = \{0,1\}^M$ and $H = \{(x_m)_{m\in M} \in G | \{m \in M | x_m = 1\}$ is finite$\}$. On G consider the topology generated by the class of the sets $\Pi_{m\in M} V_m$, where $\{m \in M | V_m \neq \{0,1\}\}$ is countable. The subgroup H is a nondiscrete, nonpseudocompact, K-group. (This answers the question proposed by J. M. Kister, Proc. Amer. Math. Soc. 13 (1962), 37.) (Received December 24, 1968.)

69T-G34. CARLOS J. R. BORGES, University of California, Davis, California 95616. On free topological groups.

M. I. Graev [Free topological groups, Transl. Amer. Math. Soc. (ser. 1) 8 (1962), 305-364] proved the following (i.e. Theorem 4): Let X be a compact Hausdorff space. A subset V of a free topological group F(X) of X is open if and only if, for each n, the sets $V_n = V \cap F_n(X)$ are open in $F_n(X)$ (where $F_n(X)$ is the set of those elements of F(X) which have length not greater than n). We prove, among other results, that Graev's theorem remains true for any Tychonoff space.
(Received December 27, 1968.)

69T-G35. ROBERT D. EDWARDS, University of Michigan, Ann Arbor, Michigan, and ROBION C. KIRBY, University of California, Los Angeles, California 95024. Deformations of spaces of imbeddings.

Let M be a compact topological manifold, and $C \subset V \subset M$ where C is compact and V is open. Let $I_C(V,M)$ be the space (with the compact-open topology) of imbeddings of V in M which restrict to the inclusion on C. Theorem. Given a neighborhood Q of the inclusion $i: V \to M$ in $I_\varphi(V,M)$, then there exists a neighborhood P such that P deforms into $I_C(V,M)$ with the deformation taking place in Q. Furthermore, the deformation is relative to M - V; that is, if $h \in P$ deforms via h_t, then $h_t | M - V = h | M - V$ for all t. Corollary 1. When C = V = M, we have the local contractibility of the group of homeomorphisms of M. Corollary 2. A locally flat isotopy $g_t : C \to M$ extends to an ambient isotopy $G_t : M \to M$. Corollary 3. Any isotopy of M can be replaced by an isotopy of small moves. Let N be the smallest nontrivial normal subgroup of the group of homeomorphisms of M. It is known that N is the subgroup generated by homeomorphisms which are the identity outside cells in M. By Corollary 3, we see that N is equal to the subgroup of homeomorphisms which are isotopic to the identity. (Received January 3, 1969.)

Miscellaneous Fields

69T-H11. J. A. MITCHEM, Western Michigan University, Kalamazoo, Michigan 49001.
The point-outerthickness and point-outercoarseness of complete n-partite graphs.

A graph G is outerplanar if it can be embedded in the plane so that each point lies on the exterior region. The point-outerthickness (point-outercoarseness) of a graph G is the minimum (maximum) number of subsets into which the point set of G can be partitioned so that the subgraph induced by each subset is outerplanar (not outerplanar). Formulae for the point-outerthickness and point-outercoarseness of the complete n-partite graphs are developed. (Received November 8, 1968.)

69T-H12. STEPHEN A. COOK, University of California, Berkeley, California 94720.
Characterizations of pushdown languages by time complexity. Preliminary report.

Theorem 1. A set A of strings is accepted by some nondeterministic stack automaton, deterministic stack automaton, writing pushdown acceptor, or two-way multihead pushdown automaton if and only if A is accepted by a deterministic Turing machine within time $T(n) = 2^{cn^2}, n^{cn}, 2^{cn}$, or n^c respectively, for some constant c. Corollary. The four classes of sets accepted by the four pushdown machines are all distinct and each is closed under union, intersection, and taking complements. Theorem 2. The deterministic and nondeterministic versions of the last two types of pushdown machines have the same computing power. For related results, see Abstract 68T-H39, these *Notices* 15 (1968), 1047. (Received December 23, 1968.)

69T-H13. ROBERT L. HEMMINGER, Vanderbilt University, Nashville, Tennessee 37203, and DONALD L. GREENWELL, Arkansas State University, State College, Arkansas. Reconstructing graphs.

The paper deals with a special case of the Ulam Conjecture which we refer to as the Terminal-Vertex Problem: If G and H are connected graphs with terminal vertices S(G) and S(H) respectively, $|S(G)| \geqq 1$, and $\sigma: S(G) \to S(H)$ is a one-to-one onto function such that $G_v \cong H_{\sigma(v)}$ for all $v \in S(G)$, then $G \cong H$ (G_v is G minus v and edges incident with v). Let G^1 be obtained from G by deleting a terminal vertex of G, and if G^n has a terminal vertex let G^{n+1} be obtained from G^n by deleting it. This leads to a subgraph $\overline{G}$ of G having no terminal vertices. Let C be the center of the cut-point block graph of $\overline{G}$; so C is either a cut-point or a block of G. The interesting case is when C is a block. For $v \in C$ let a branch of G at v be the maximal connected subgraph of G containing v but no other vertices of C. We characterize the counterexamples to the Terminal-Vertex Problem having at most two branches that contain terminal vertices of G and then we prove the Theorem. If G has at least three branches that contain terminal vertices of G and if C is a circuit then the Terminal-Vertex Problem is true for G and H. This paper also contains a summary of most of the known results on the Ulam Conjecture as well as a number of new results on the corresponding edge problem. (Received December 23, 1968.)

69T-H14. JOHN W. HEIDEL, University of Tennessee, Knoxville, Tennessee 37916.
Uniqueness, continuation and nonoscillation for a second order nonlinear differential equation.

The equation considered is (1) $y'' + f(t)y^{\gamma} = 0$ where $f(t)$ is nonnegative and continuous on $[0,\infty)$ and γ is the quotient of odd positive integers. Coffman and Ullrich (On the continuation of solutions of a certain nonlinear differential equation, Monatsh. Math. 71 (1967), 5, 395-392) have shown that if, in addition, $f(t)$ is positive and locally of bounded variation on $[0,\infty)$ and $\gamma > 1$, then all solutions can be continued to $[0,\infty)$. We give an analogous result for $0 < \gamma < 1$. Definition. A nontrivial solution $y(t)$ of (1) which satisfies $y(t_0) = y'(t_0) = 0$ for some $t_0 \geqq 0$ is called singular. If (1) has no singular solution then the zero solution of (1) is unique. Theorem 1. If $0 < \gamma < 1$ and $f(t)$ is positive and class C' on $[0,\infty)$ then the zero solution of (1) is unique. The result of Coffman and Ullrich and Theorem 1 are then both improved by allowing $f(t)$ to have isolated zeros. The idea is to transform a noncontinuable or singular solution of (1) into an oscillatory solution of an equation of the form of (1) and then apply a nonoscillation theorem. (Received December 23, 1968.)

69T-H15. JAMES B. TURNER, Stanford Research Institute, Menlo Park, California 94025.
Trivalent graphs without disjoint 1-factors.

Let $G = (V,E)$ be a graph which is regular of degree three. A 1-factor F of G is a subset of edges of G such that (a) no two edges of F are incident and (b) every point of the graph appears as an endpoint of an edge of F. A pair of 1-factors of a graph is disjoint if they have no common edge. Theorem. Let G be a trivalent graph such that every pair of its 1-factors has exactly one edge in common. Then G is nonplanar. (Received December 23, 1968.)

69T-H16. SURJEET SINGH and KAMLESH WASAN, University of Delhi, Delhi 7, India.
Selfinjective rings and multiplication rings.

All the rings considered here are commutative and with unity. A ring R is said to have (I)-property if each proper homomorphic image of R is a selfinjective ring. Noetherian commutative rings with (I)-property were characterised by Levy. A ring R is said to be a multiplication ring if for every pair of ideals A, B of R with $A \subseteq B$, there exists an ideal C of R such that $A = BC$. Following results have been established. (i) Let R be a noetherian commutative ring with unity. Then for each proper prime ideal P, R_P has (I)-property iff R is a finite direct sum of rings R_i, each of which has (I)-property. (ii) A noetherian commutative ring R with unity is a multiplication ring iff for each proper prime ideal P, R_P is a ring with (I)-property and R contains at most one minimal ideal. (Received December 27, 1968.)

69T-H17. D. S. ORNSTEIN, Ohio State University, Columbus, Ohio 43210, and LOUIS SUCHESTON, Stanford University, Stanford, California 94305. On the ergodic theorem and infinite invariant measures for operators. Preliminary report.

If T is a positive linear operator on L_1 of a σ-finite measure space, satisfying the condition $\sup_n \|T^n\|_1 < \infty$, then the space decomposes into the 'remaining' part Y and the 'disappearing' part Z; see Z. Wahrscheinlichkeitstheorie vol. 8, pp. 1-11 and 353-356, where the following question is asked: If Z is the entire space, does $T^n f$, $f \in L_1$, converge Cesaro at each point? It is shown by example that (1) the answer is negative. Furthermore (2) there need not exist any unbounded function ('infinite invariant measure') e such that $T^*e = e > 0$ on Z = the entire space. Related results are: (3) In the theorem of Harris about existence of a σ-finite invariant measure (Proc. Third Berkeley Sympos. Math. Stat. Prob., vol. 2, 113-124) the assumption that $\|T\| = 1$, i.e., the operator is Markovian, can be weakened to $\sup_n \|T^n\| < \infty$ and (4) in the Dunford-Schwartz ergodic theorem the assumption $\|T\|_\infty \leq 1$ may be weakened to $\sup_n \|T^n\|_\infty < \infty$. (Received December 30, 1968.)

69T-H18. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. Independence of the axiom of extensionality.

Let $(M, \in)$ be a model for the set-theoretical axioms of extensionality, replacement, power-set, sum-set, infinity, choice and regularity. Let M' denote M with the exclusion of the empty set $\emptyset$. Consider the model $(M', \in')$ where $x \in' y$ if and only if $x \in y$ and $x \neq \emptyset$. Proposition. $(M', \in')$ is a model for the abovementioned set-theoretical axioms except for the axiom of extensionality (which states: equal sets are elements of the same sets). (Received January 2, 1969.)

69T-H19. H. JEROME KEISLER, University of Wisconsin, Madison, Wisconsin 53706. Categorical theories in infinitary logic.

A sentence s of the infinitary logic $L_{\omega_1 \omega}$ is said to be α-categorical iff any two models of s of power α are isomorphic. In what follows, the notions of elementary extension and α-homogeneous model are in the sense of ordinary finitary logic. Theorem 1. Let $\alpha > \omega$ and let s be a sentence of $L_{\omega_1 \omega}$ of the form $\bigwedge_{m<\omega} \forall x_1 \ldots x_{p(m)} \bigvee_{n<\omega} s_{mn}$, where each s_{mn} is finite. Assume (1) s is α-categorical; (2) every countable model of s has arbitrarily large elementary extensions which are models of s; (3) every model of s of power α is ω_1-homogeneous. Then for every $\beta > \omega$, s is β-categorical and every model of s of power β is β-homogeneous. The proof uses methods employed by Morley and Ressayre for finitary logic. Theorem 2. If either $\alpha = \omega_1$ or $\beta = \beth_\gamma$ where $cf(\gamma) > \omega$, then Theorem 1 holds without the hypothesis (2). Analogous results hold for sentences s having countably many models of power α (up to isomorphism). The question whether Theorem 1 holds without hypotheses (2) or (3) is open. (Received December 20, 1968.)

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Form Approved, Budget Bureau No. 46-R029

1. DATE OF FILING: September 24, 1968
2. TITLE OF PUBLICATION: Notices of the American Mathematical Society
3. FREQUENCY OF ISSUE: 8 times a year
4. LOCATION OF KNOWN OFFICE OF PUBLICATION *(Street, city, county, state, ZIP code)*: 321 South Main Street, P. O. Box 6248, Providence, Rhode Island 02904
5. LOCATION OF THE HEADQUARTERS OR GENERAL BUSINESS OFFICES OF THE PUBLISHERS *(Not printers)*: Same
6. NAMES AND ADDRESSES OF PUBLISHER, EDITOR, AND MANAGING EDITOR

PUBLISHER *(Name and address)*: American Mathematical Society, 321 South Main Street, Providence, Rhode Island 02904

EDITOR *(Name and address)*: Everett Pitcher, Lehigh U., Bethlehem, Pa. 18015 - G. L. Walker, 321 So. Main St., Providence R. I. 02904

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POD Form 3526 May 1960

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AMERICANA OF NEW YORK
April 2-5, 1969

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